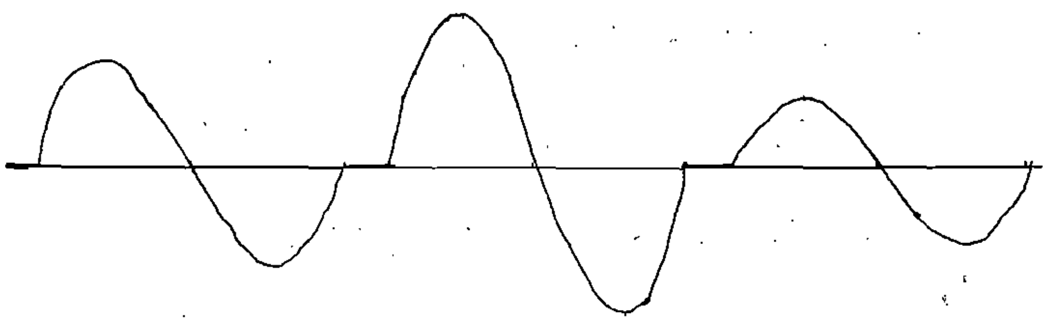
DSB-Modulation

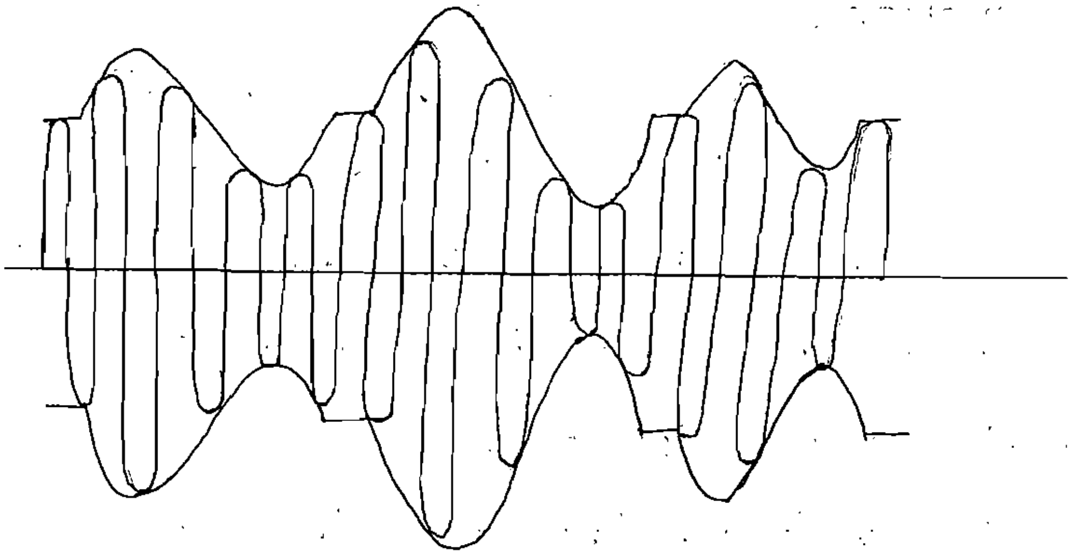
Introduction :-

When the carrier is amplitude modulated by a single sine wave the resulting signal consists of three frequencies i.e... Original carrier and two side bands. This system is known as Double side-band full carrier (DSBFc).

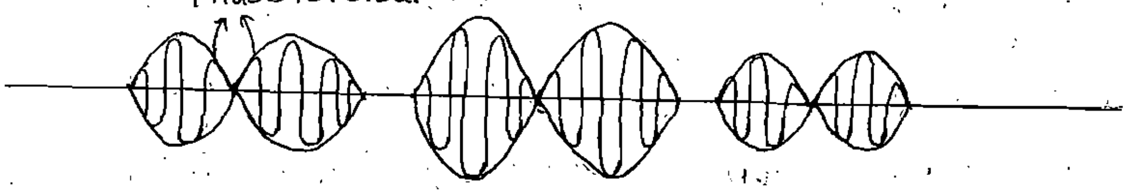
Generally we know that the carrier signal does not convey any information. The real information is conveyed by two side bands. But two third of power is wasted to transmit the carrier signal. So we can supress the carrier and remaining signal contains only lower and upper side bands. Then the signal is called Double side band supressed carrier (DSBsc). Now the power wasted for carrier can be put in to the side bands, to transmit for longer distances.



(a) modulating signal



(b) AM wave



(c) Supressed carrier wave.

Time domain description :=

When the carrier is supressed during the modulation process, the resulting is only the algebraic sum of upper and lower side bands. The equation of DSBSC AM signal is given as

$$s(t) = m(t)\, c(t)$$

$$= A_m \cos 2\pi f_m t \cdot A_c \cos 2\pi f_c t$$

$$= \frac{A_m A_c}{2}\left[\cos 2\pi (f_c + f_m)t + \cos 2\pi (f_c - f_m)t\right]$$

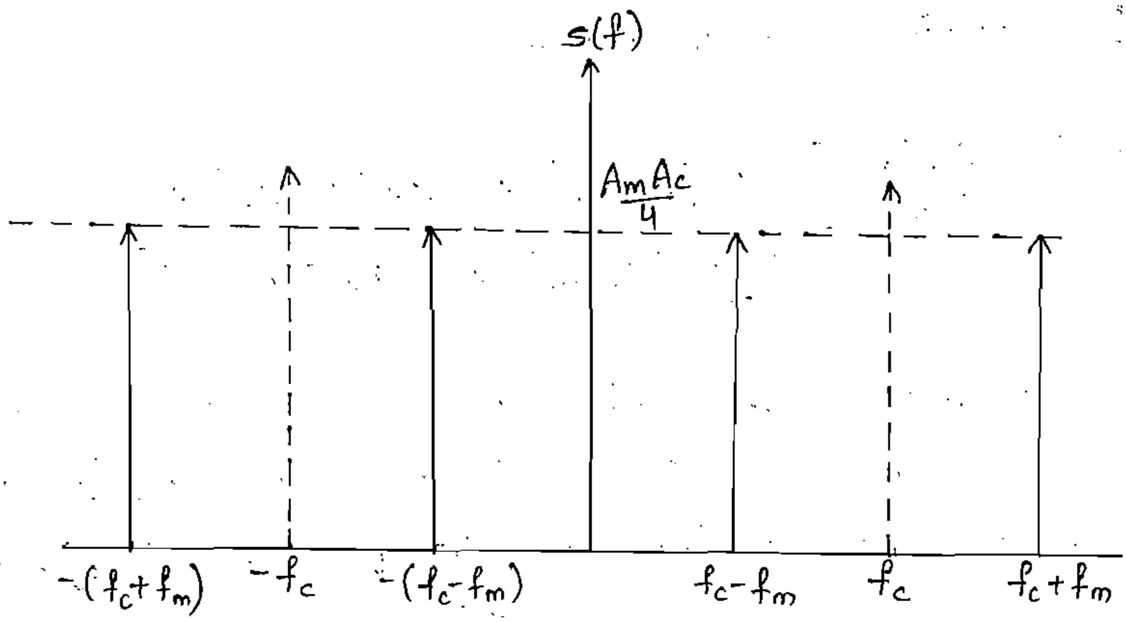
$f_c + f_m$ is upper side band ; $f_c - f_m$ is lower side band.

Frequency domain description :=

In DSBSC, as the carrier is supressed it is denoted by dotted line. The spectrum space occupied by DSB is $2f_m$ or 2ω, which is same as that for a conventional AM signal.

From time domain description we have

$$s(t) = \frac{A_c A_m}{4}\Big[\delta(f-(f_c+f_m) + \delta(f+(f_c+f_m) + \delta(f-(f_c-f_m) + \delta(f+(f_c-f_m)\Big]$$



Spectrum of DSBSC modulated wave

Band width :=

The spectrum space occupied by DSB-SC is $2f_m$, so its band width is $2f_m$.

Single tone modulation :=

If the modulating signal contains only one frequency then it is called single tone modulation.

The sinusoidal modulating signal is given by

$$m(t) = A_m \cos 2\pi f_m t.$$

There fore, DSBSC modulated wave is given by

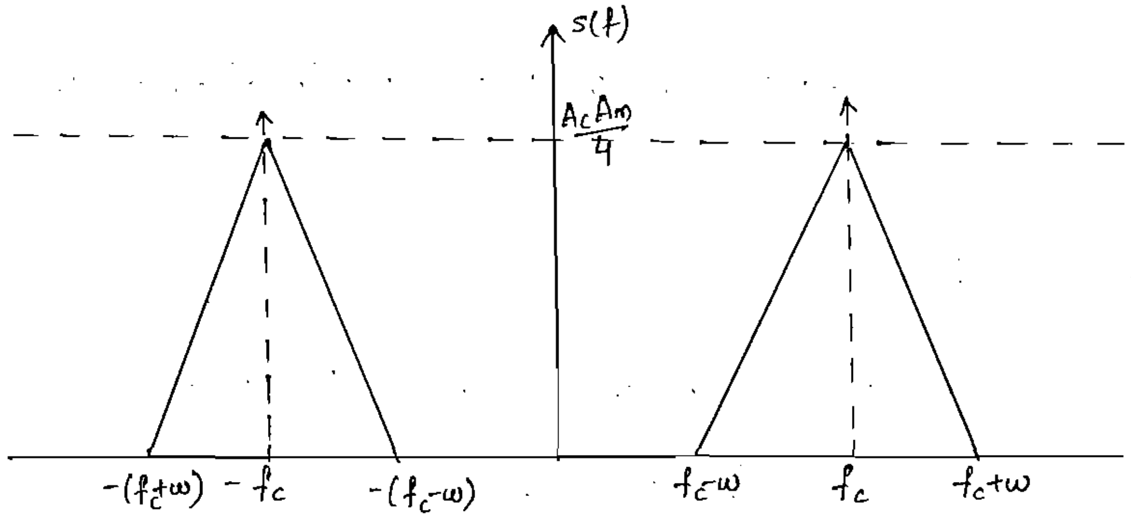
$$s(t) = A_c A_m \cos 2\pi f_c t \cos 2\pi f_m t$$

$$= \frac{1}{2} A_c A_m \left[\cos 2\pi (f_c + f_m) t + \cos 2\pi (f_c - f_m) t\right]$$

By applying fourier transform

$$S(f) = \frac{A_c A_m}{4} \Big[\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m)) + \delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m)) \Big]$$

We can observe that the spectrum of DSBSC modulated wave for the case of a sinusoidal modulating wave consists of delta functions located at $f_c \pm f_m$ & $-f_c \pm f_m$



spectrum of single tone AM-SC

Power :=

As the carrier is absent the total power is only due to side bands.

$$P_{DSBFC} = P_c + P_{SC} = P_c + P_c \frac{\mu^2}{2}$$

Carrier is supressed so

$$P_{DSBSC} = P_c \frac{\mu^2}{2}$$

$$\%\text{ power saving in DSBSC} = \frac{P_{DSBFC} - P_{DSBSC}}{P_{DSBFC}} \times 100$$

$$= \frac{P_c\left(1+\frac{\mu^2}{2}\right) - P_c\frac{\mu^2}{2}}{P_c\left(1+\frac{\mu^2}{2}\right)} \times 100$$

$$= \frac{2}{2+\mu^2} \times 100.$$

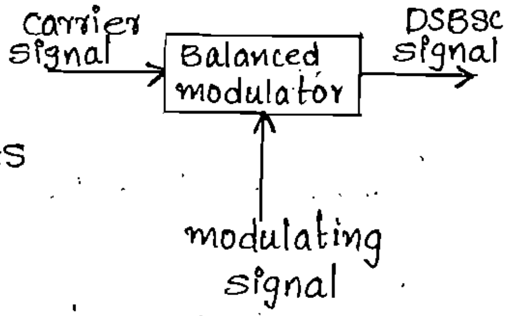
if $\mu = 1$ then % power saving in DSBSC is 66.6%.

Generation of DSBSC wave :=

For generating a DSBSC signal we have seen that m(t) and c(t) should get multiplied with each other. So this type of can be obtained by using a product modulator or balanced modulator.

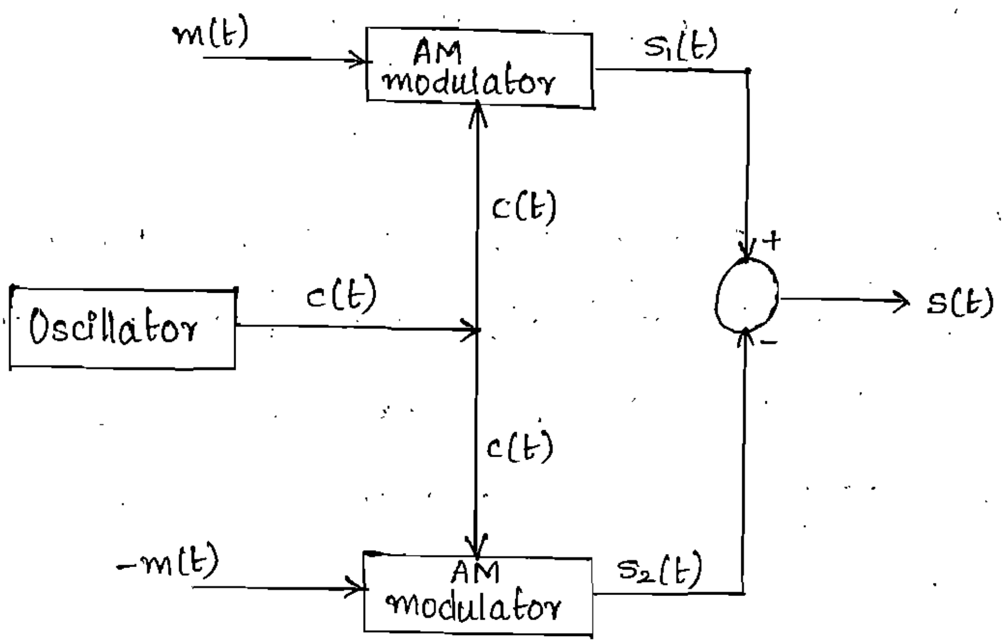
Balanced Modulator :=

The balanced modulator is used to supress the carrier from the AM signal. The inputs are carrier and modulating signal. The out puts are upper and lower side bands.



Principle used in balanced modulator :=

When two signals at different frequencies are passed through a non linear resistance, the AM signal is generated with supressed carrier. The non linear devices such as diode, JFET and transistor are used in balanced modulator.



The balanced modulator circuit consists of two standard amplitude modulators which are arranged in balanced configuration, to supress the carrier. The two modulators are identical except for the sign reversal of the modulating wave applied to the input of one of them.

The outputs of modulators can be given as

$$s_1(t) = A_c(1 + k_a m(t)) \cos 2\pi f_c t$$

$$s_2(t) = A_c(1 - k_a m(t)) \cos 2\pi f_c t$$

$$s(t) = s_1(t) - s_2(t)$$

$$= A_c \cos 2\pi f_c t \left[1 + k_a m(t) - 1 + k_a m(t)\right]$$

$$= A_c \cos 2\pi f_c t \left(2 k_a m(t)\right)$$

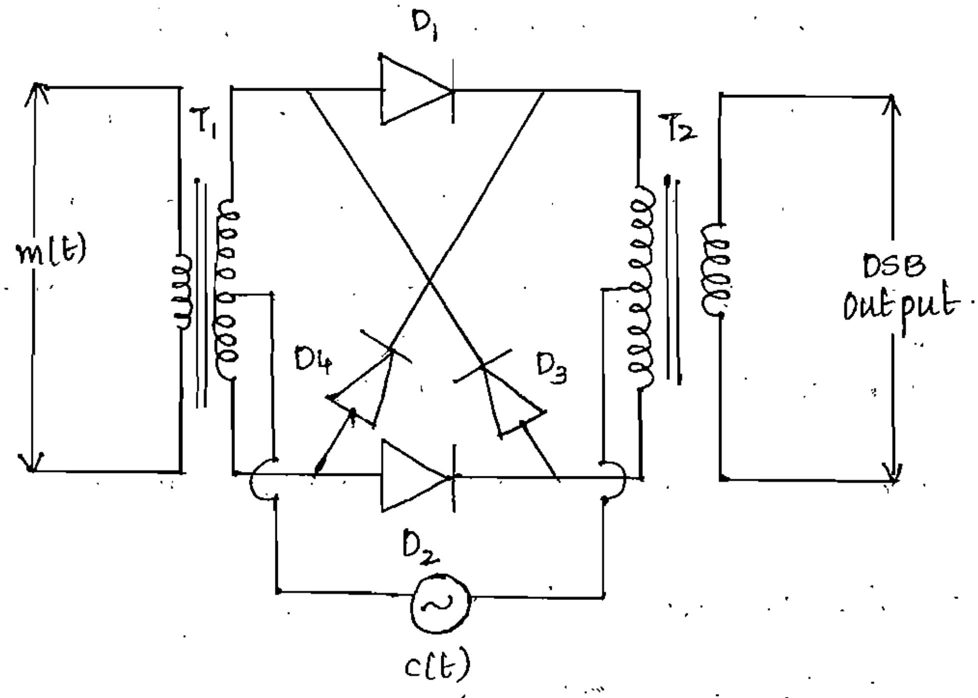
$$= 2 k_a m(t) \, c(t).$$

The balanced modulator output is equal to the product of modulating and carrier.

There are two types of balanced modulators.

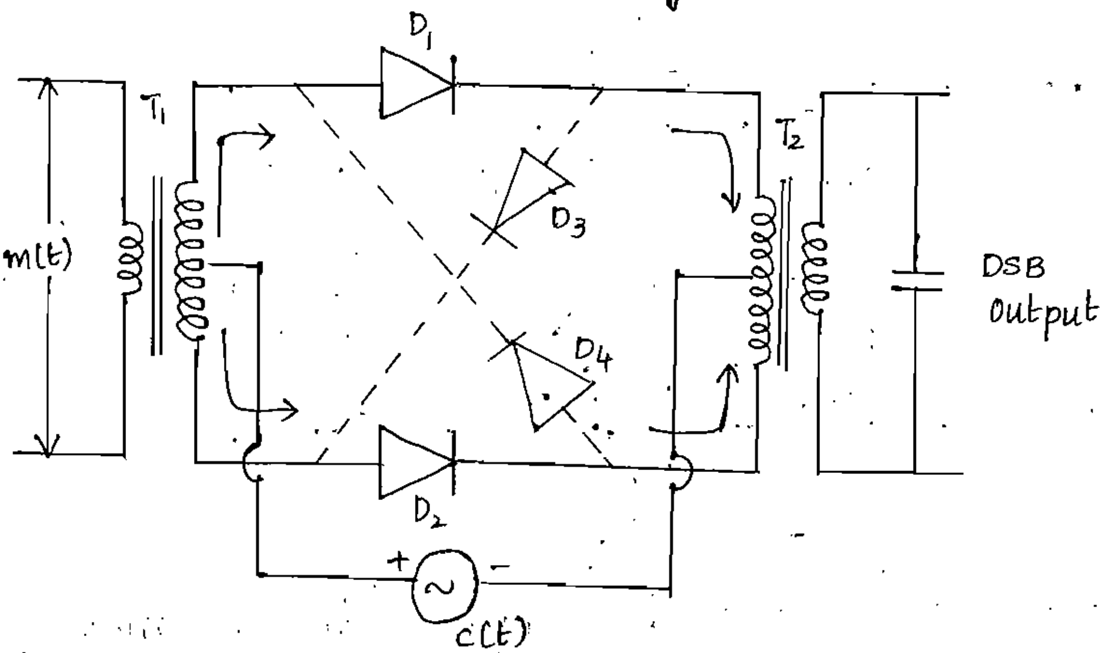
* Diode or Ring or Lattice modulator.
* JFET modulator.

Diode Modulator :-



Diode modulator consists of an input transformer T_1 and an output transformer T_2 and four diodes connected in a bridge circuit. In this the carrier signal is applied to center taps of input and output transformers and modulating signal is applied to input transformer T_1. The output appears across secondary of the transformer. The diodes connected in the bridge acts like switches and are controlled by carrier signal because it is higher in frequency and amplitude than the modulating signal.

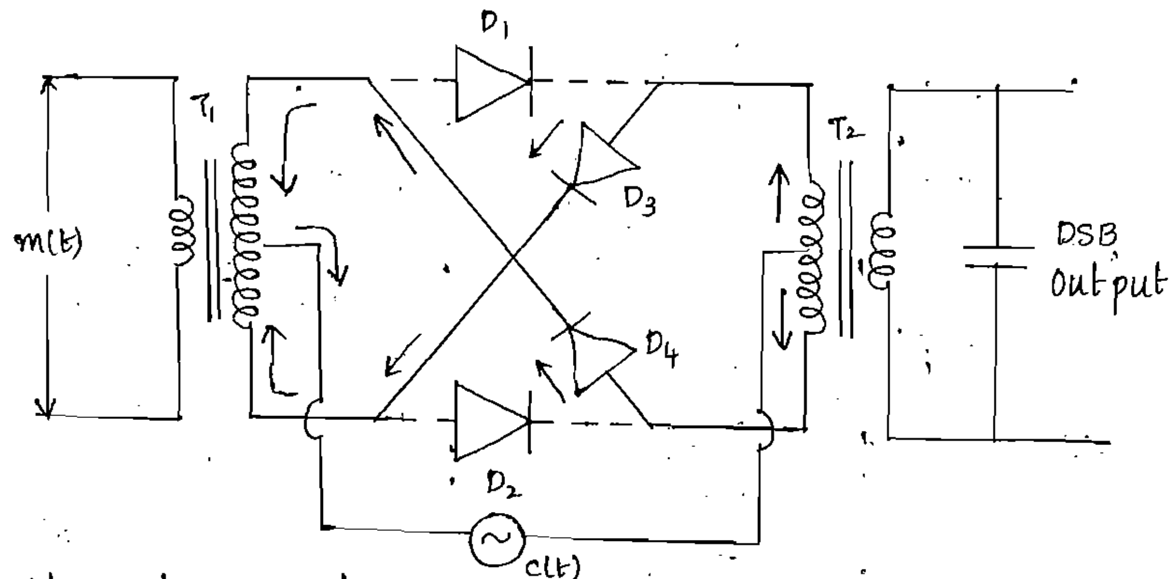
Positive half cycle of carrier signal:-



In this we assume that the modulating input is zero. In positive half cycle of carrier signal, the diodes D_1 and D_2 are forward biased and D_3 and D_4 are reverse biased. The current divides equally in upper and lower portions of primary winding of T_2. The current produce magnetic field which is equal and opposite in upper and lower part of the winding. Hence they cancel each other and produce no output at secondary of T_2. Thus the carrier is supressed.

Negative half of carrier signal:-

In negative half of carrier signal the diodes D_1 and D_2 are reverse biased and D_3 and D_4 are forward biased.



Similar to positive half cycle, the magnetic fields in primary winding of T_2 are equal and opposite canceling each other. There fore output at T_2 secondary is zero.

The supression of carrier in both the half cycles depends on diode characteristics and exactness of center tap of transformer to give exactly equal upper and lower currents and magnetic fields.

With modulating signal:-

Before we assumed that the modulating input is zero. But now consider a low frequency sine wave is applied to primary of T_1 as modulating signal. This will appears across T_1 secondary. The diodes D_1 and D_2 are forward biased and they connect secondary of T_1 to primary ~~of T_2. As a result the modulating~~ signal is applied to primary of T_2. In negative half cycle, the diodes D_3 and D_4 are forward biased. athey connect secondary of T_1 to primary of T_2 with reverse connections which results in 180° phase shift in modulated signal.

This balanced modulator works perfectly when the carrier signal is square wave

The expression can be given as

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos 2\pi f_c t(2n-1).$$

$$s(t) = c(t) \cdot m(t)$$

$$= \left(\frac{4}{\pi} \cos 2\pi f_c t - \frac{4}{3\pi} \cos(3 \times 2\pi f_c t) + \cdots\cdots\right) m(t)$$

$$= \frac{4}{\pi} \cos 2\pi f_c t \, m(t) - \frac{4}{3\pi} \cos(3 \times 2\pi f_c t)\, m(t) + \cdots\cdots$$

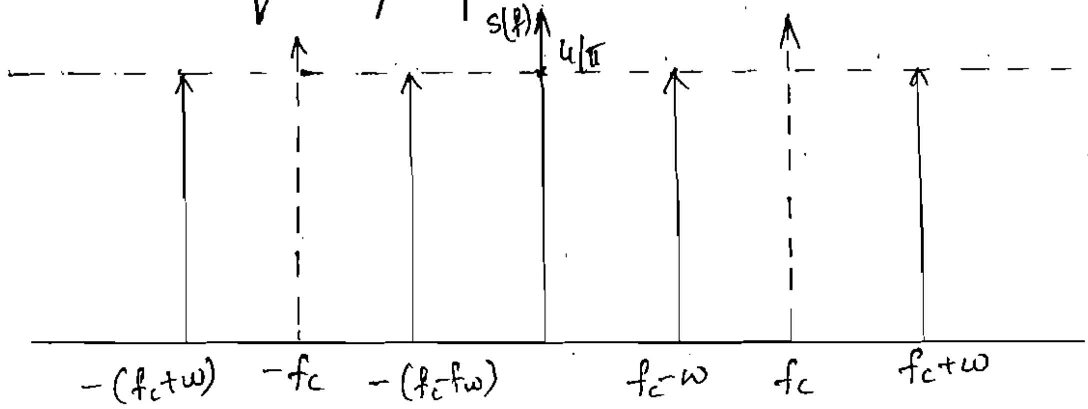
Apply fourier transforms.

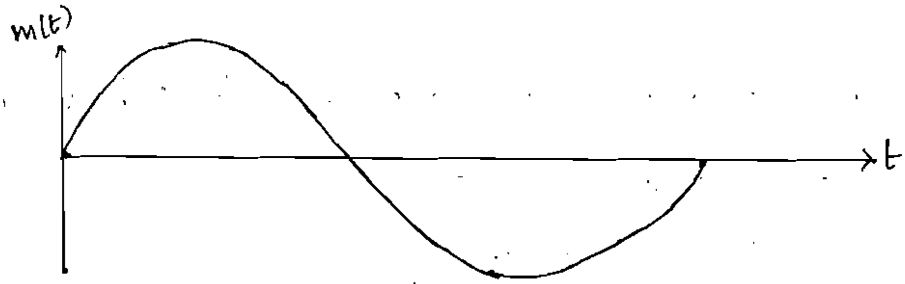
$$s(f) = \frac{4}{\pi}\left[M(f-f_c) + M(f+f_c)\right] - \frac{4}{3\pi}\left[M(f-3f_c) + M(f+3f_c)\right] + \cdots$$

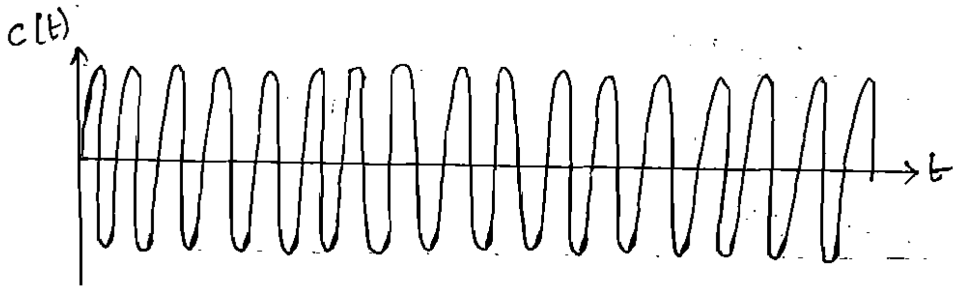
Eliminate high frequency terms then we get

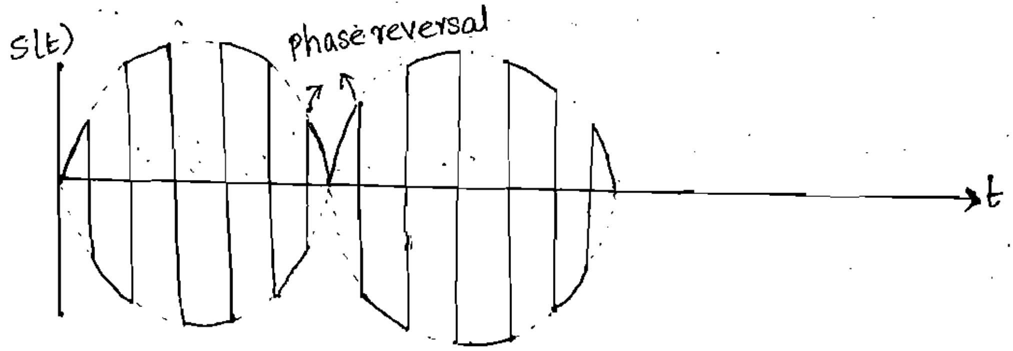
$$s(f) = \frac{4}{\pi}\left[M(f-f_c) + M(f+f_c)\right].$$

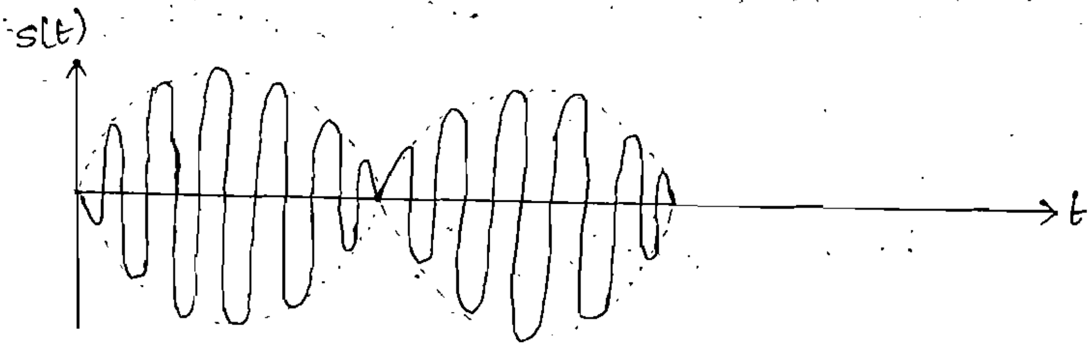
Then the frequency spectrum is



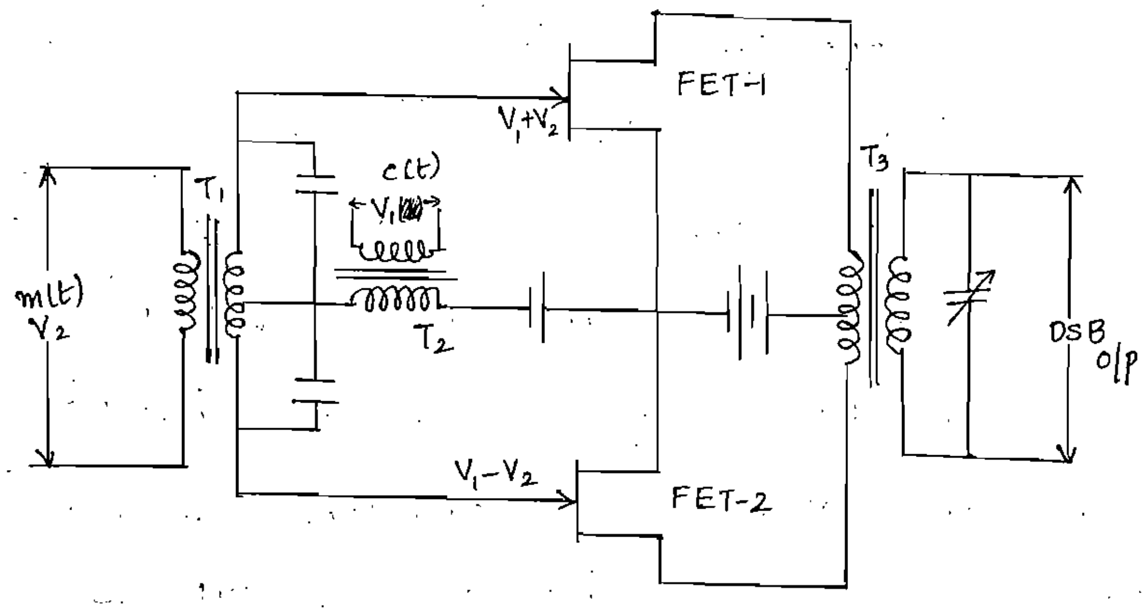








JFET balanced Modulator :-



This modulator consists of input transformers T_1 and T_2, an output transformer T_3 and two FETs. Here the carrier signal is applied to centre taps of input T_1 and output T_3 through transformer T_2. The modulating signal is applied to input transformer T_1. The carrier voltage is applied to two ~~taq~~ gates in phase where as modulating voltage appears 180° out of phase, at the gates, since they are at the opposite ends of center tapped transformer.

In the absence of modulating signal, the FET currents due to carrier signal are equal in amplitude but opposite in direction and cancel each each other resulting in no output of secondary of Transformer. Thus carrier is supressed.

When modulating signal is applied, the FET currents due are due to carrier as well as the modulating signal. The FET currents due to carrier are equal and opposite and cancel eachother. As modulating signal is applied 180° out of phase at the gate, the FET currents are equal but not opposite. Hence they do not cancel each other. Therefore the out put is two side bands with supressed carrier. The supression of carrier depends on symmetry of circuit components.

Mathematical Expression for balanced modulator:=

The transfer characteristics of FET (I_D, V_{GS}) can be given as $I_D = I_0 + aV_{gs} + bV_{gs}^2 + \cdots$

Here I_0 is maximum drain current when $V_{GS} = 0$. a and b are constants.

From this $I_{D_1} = I_0 + aV_{gs_1} + bV_{gs_1}^2$

$$I_{D_2} = I_0 + aV_{gs_2} + bV_{gs_2}^2$$

As the drain currents are in opposite direction the current through primary of coil of T_3 can be given as $I_P = I_{D_1} - I_{D_2}$

$I_p = a(V_{gs_1} - V_{gs_2}) + b(V_{gs_1}^2 - V_{gs_2}^2)$

Here $V_{gs_1} = V_1 + V_2$ and $V_{gs_2} = V_1 - V_2$

$I_p = a(2V_2) + b(4V_1V_2)$.

We know that $V_1 = c(t) = A_c \cos\omega_c t$

$V_2 = m(t) = A_m \cos\omega_m t$

$I_p = 2a A_m \cos\omega_m t + 4b A_c A_m \cos\omega_c t \cos\omega_m t$.

$= 2a A_m \cos\omega_m t + 2b A_c A_m [\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t]$

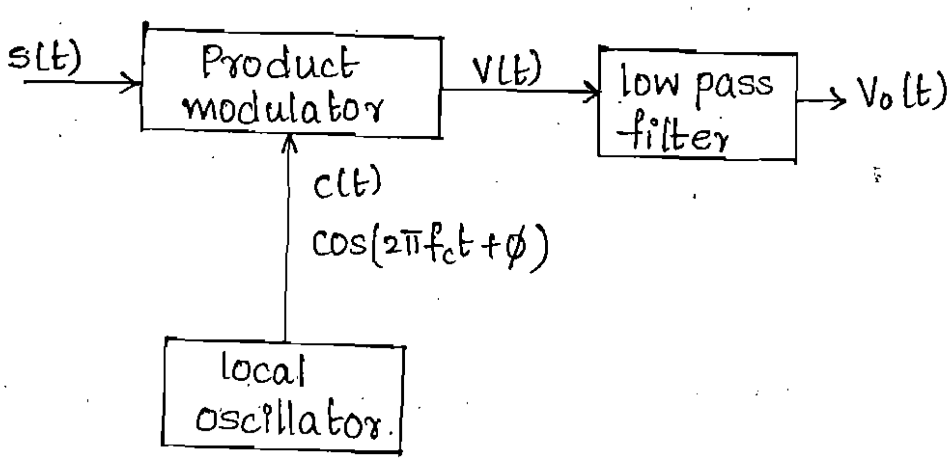
$2a A_m \cos\omega_m t$ is message signal.

$\omega_c - \omega_m$ is lower side band

$\omega_c + \omega_m$ is upper side band.

From the above expression we can say that the carrier is supressed.

Coherent detection of DSBSC modulated wave:=



The modulating signal m(t) is recovered from a DSBSC wave s(t) ; by multiplying s(t) with a locally generated sinusoidal wave and then passing through a low pass filter.

For faith ful recovery of message signal m(t) the output of local oscillator should be exactly coherent or synchronized in both frequency and phase with carrier wave c(t) which is used to generate s(t). This method of demodulation is called coherent or synchronous detection.

The output of product modulator is given as

$$V(t) = s(t)\cos(2\pi f_c t + \phi)$$

$$= A_c \cos 2\pi f_c t \cdot \cos(2\pi f_c t + \phi)\, m(t).$$

$$= \frac{1}{2} A_c \cos\phi\, m(t) + \frac{1}{2} A_c \cos(4\pi f_c t + \phi)\, m(t).$$

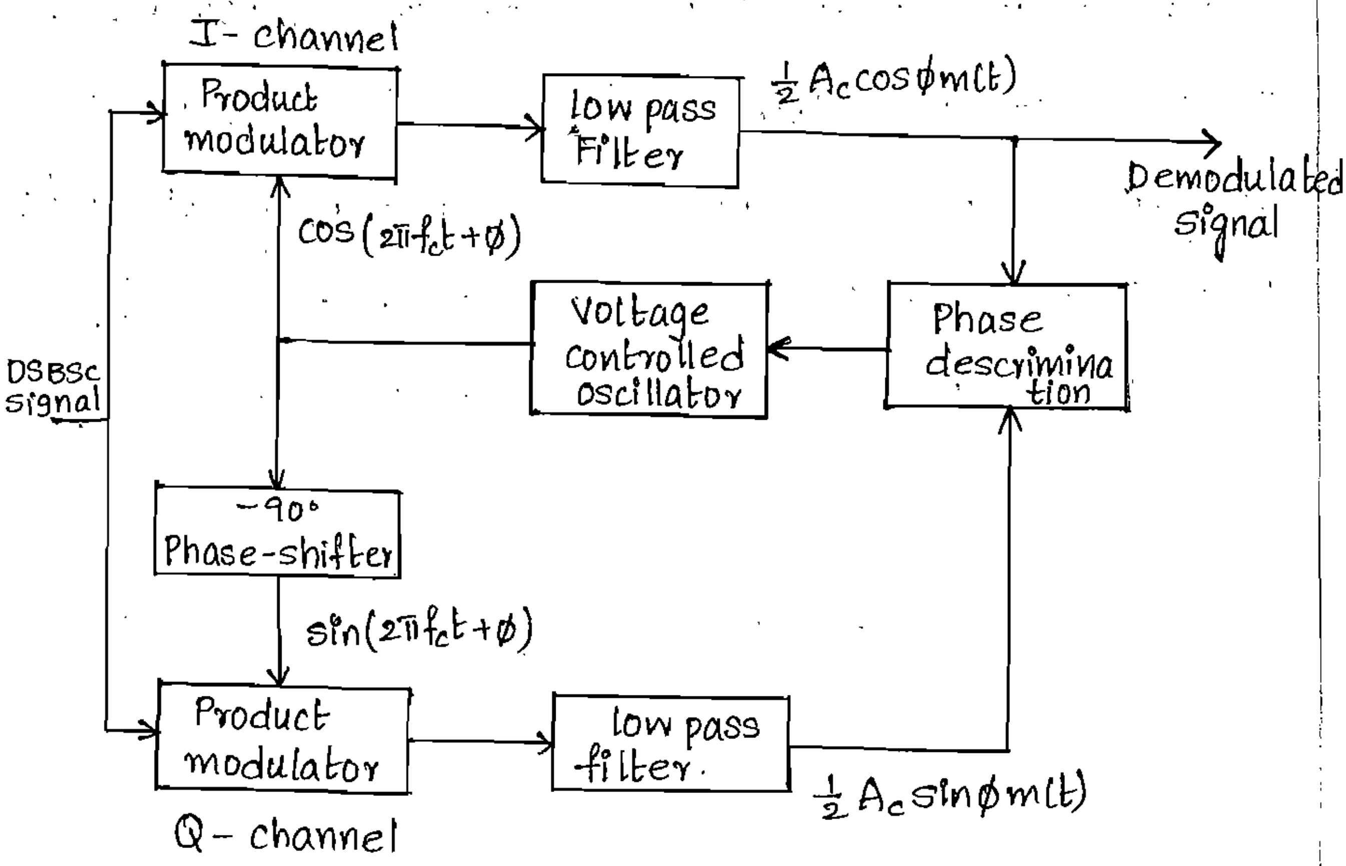
The out put consists of two terms. They are scaled version of message signal and unwanted term. The Unwanted term is removed by passing through low pass filter.

The output is $V_o(t) = \frac{1}{2} A_c \cos\phi\, m(t)$.

When phase error ϕ is constant, the demodulated signal $V_o(t)$ is proportional to m(t). It is maximum when $\phi = 0$ and minimum when $\phi = \pm \pi/2$. The zero demodulated signal which occurs when $\phi = \pm\pi/2$ is called quadrature null effect.

Because of phase error ϕ the detector output is attenuated by a factor $\cos\phi$. If $\cos\phi$ is constant the detector output gives undistorted modulating signal. But practically $\cos\phi$ is not constant. So to maintain the local oscillator in perfect synchronism we use costas loop.

Costas loop :=



The coastas loop is nothing but the negative feedback system designed to ~~main~~ maintain the local-oscillator synchronous with carrier wave.

It consists of two coherent product modulators supplied with same input signal. The local oscillator signal supplied to product modulators are 90° out of phase. The frequency of local oscillator is adjusted to be same as carrier frequency f_c. The product modulator in the upper path is called "in phase coherent detector or I-channel." where as, which is in lower path is called "quadrature phase detector or Q-channel." The outputs of these two channels are given to phase descriminator which consists of a multiplier followed by a low pass filter, produces a dc control signal proportional to phase error ϕ. This dc control signal is used to correct the phase error in local oscillator.

SSB Modulation

Introduction:=

When the carrier is amplitude modulated by a single sine wave, the resulting signal consists of three frequencies i.e. Original carrier and two side bands ($f_c \pm f_m$). In normal AM system both the side bands and carrier are transmitted. This is known as DSBFC. But we know that the carrier signal does not convey any information. so we can supress the carrier and we get DSBSC.

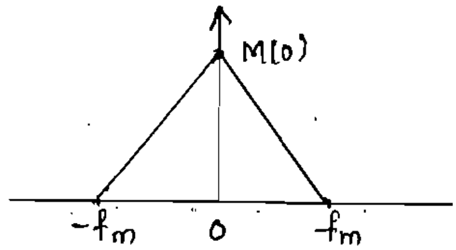
As the two side bands carry same information there is no need to transmit both the side bands to convey information. One side band may be supressed. The resulting signal is single side band known as single side band supressed carrier (SSB).

Definition := "A modulation process in which the modulated signal contains no carrier component and has only one side band is called single side band modulation or SSB modulation"

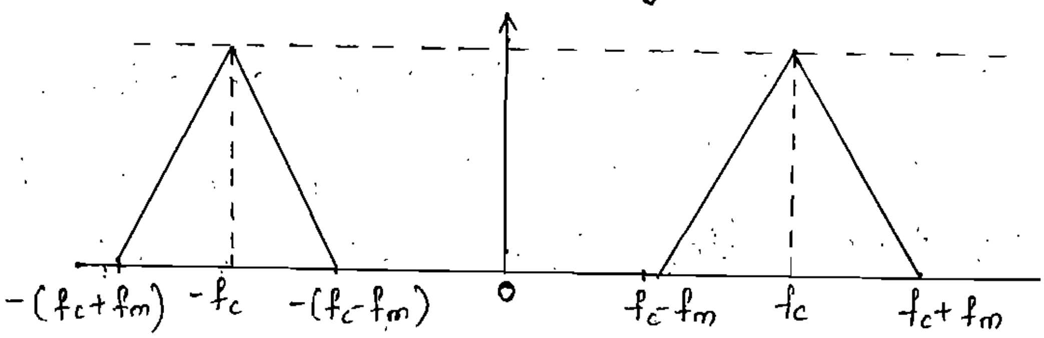
So the power which is wasted to transmit the carrier signal and a side band, can be put in to a single side band for stronger signals over longer distances.

Frequency domain description :=

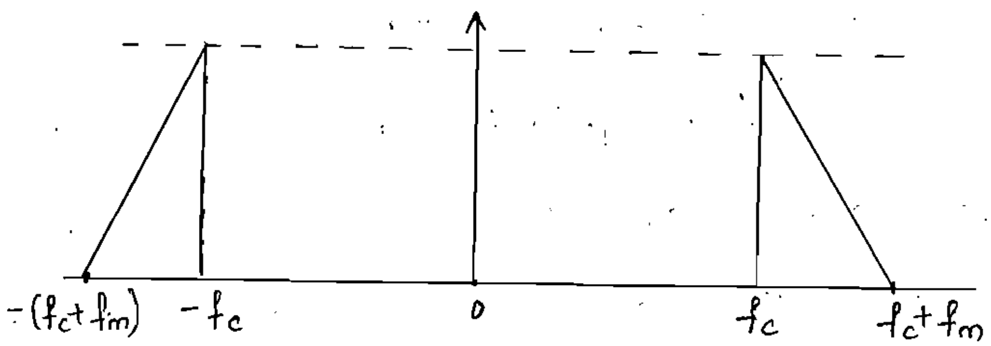
The frequency domain display of single side band modulated wave depends on the side band which is transmitted. The spectrum of DSBSC modulated wave can be obtained by multiplying m(t) and c(t).



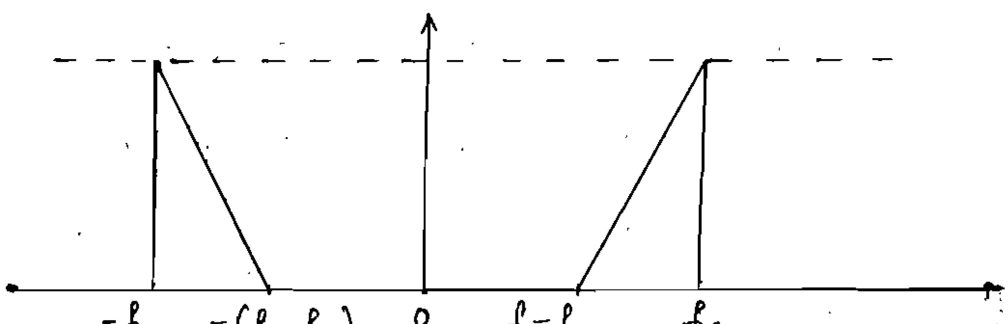
Spectrum of message signal



Spectrum of DSBSC wave



Spectrum of SSB with upper sideband



lower sideband.

Advantages of SSB :=

* The spectrum space occupied by SSB signal is f_m which is only half that of AM and DSB. This reduction in frequency or band width allows more signals to transmit in same frequency range.
* Due to supression of carrier and one side band, power is saved.
* SSB signal has less band width than an AM or DSB so there will be less noise on it.
* Fading does not occur in SSB.

Disadvantages :=

* The generation and reception of SSB signal is a complex process.
* SSB is not used for transmission of high frequency signals such as music signals.

Power relations in SSB wave :=

% power saving w.r.to DSBFC.

We know that total power transmitted in AM is

$$P_t = P_c + P_{USB} + P_{LSB}$$

$$= \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}$$

$$= \frac{A_c^2}{R}\left[1+\frac{\mu^2}{2}\right] = P_c\left(1+\frac{\mu^2}{2}\right).$$

If the carrier and one side band is supressed, then the total power transmitted in SSB-SC is

$$P_{SSB} = P_{LSB} = P_{USB}$$

$$P_{SSB} = \frac{\mu^2 A_c^2}{8R} = P_c\left(\frac{\mu^2}{4}\right).$$

Power saved due to transmitting only one side band component is given by

$$\%\text{ power saving} = \frac{P_t - P_{SSB}}{P_t} \times 100$$

$$= \frac{P_c\left(1+\frac{\mu^2}{2}\right) - \frac{\mu^2}{4} P_c}{P_c\left(1+\frac{\mu^2}{2}\right)} \times 100$$

$$= \frac{1+\frac{\mu^2}{2} - \frac{\mu^2}{4}}{1+\frac{\mu^2}{2}} \times 100$$

$$= \frac{4+\mu^2}{4+2\mu^2} \times 100.$$

if $\mu = 1$ then

$$\%\text{ power saving} = \frac{5}{6} \times 100 = 83.33\%.$$

Thus 83.33% power is saved due to supression of carrier wave and one side band.

% power saving w.r.to DSB-SC system :=

We know that the total power transmitted in DS DSB-SC is

$$P_{DSBSC} = P_{LSB} + P_{USB}$$

$$= \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R} = P_c \frac{\mu^2}{2}$$

total power transmitted in SSB-SC is

$$P_{SSB} = P_{LSB} = P_{USB}$$

$$P_{SSB} = \frac{A_c^2 \mu^2}{8R} = P_c \frac{\mu^2}{4}$$

$$\%\text{ power saving} = \frac{P_{DSBSC} - P_{SSB}}{P_{DSBSC}} \times 100$$

$$= \frac{P_c \frac{\mu^2}{2} - P_c \frac{\mu^2}{4}}{P_c \frac{\mu^2}{2}} \times 100$$

$$= \frac{\mu^2}{2} \times 100$$

if $\mu = 1$ then

$$\%\text{ power saving} = \frac{1}{2} \times 100 = 50\%$$

Thus 50% of power is saved due to supression of one side band from DSB-SC signal.

Time domain description :=

The SSB signal is generated by passing a DSBSC modulated wave through a band pass filter having a transfer function $H_u(f)$.

We know that $S_{DSBSC}(t) = A_c m(t) \cos(2\pi f_c t)$.

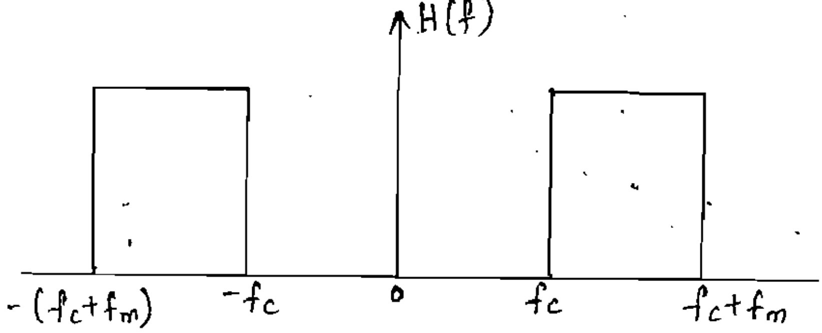
The low pass complex envelope of DSBSC modulated wave is expressed as

$$\tilde{S}_{DSBSC}(t) = A_c m(t).$$

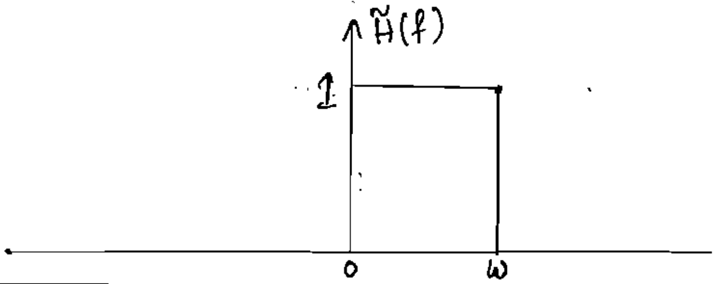
Let $s_u(t)$ be the SSB wave which has only upper-side band; and $\tilde{s}_u(t)$ be the complex envelope of $s_u(t)$.

$$s_u(t) = Re\left[\tilde{s}_u(t)\, e^{j2\pi f_c t}\right]$$

The frequency response of ideal band pass filter is.



Replace transfer function $H_u(f)$ by an equivalent low-pass filter transfer function $\tilde{H}_u(f)$.



Where $\tilde{H}_u(f) = \frac{1}{2}(1 + sgn(f)) \quad 0 < f < \omega$

$= 0 \quad$ otherwise.

the spectrum of complex envelope of DSBSC modulated wave $\tilde{s}_{DSBSC}(t)$ is

$$\tilde{S}_{DSBSC}(f) = A_c M(f).$$

Now $\tilde{S}_u(f) = \tilde{H}(f)\, \tilde{S}_{DSBSC}(f)$

$$= \frac{1}{2}(1 + sgn(f))\, A_c M(f).$$

$$= \frac{A_c}{2}\left[M(f) + M(f)\, sgn(f)\right].$$

from the definition of Hilbert transform

We have $\hat{m}(t) = -j\, sgn(f)\, M(f).$

$$\therefore \tilde{S}_u(f) = \frac{A_c}{2}\left[M(f) + j\hat{m}(t)\right]$$

$$\tilde{S}_u(t) = \frac{A_c}{2}\left[m(t) + j\hat{m}(t)\right].$$

We have $S_u(t) = Re\left[\tilde{S}_u(t)\, e^{j2\pi f_c t}\right].$

$$S_u(t) = Re\left[\frac{A_c}{2}\left(m(t) + j\hat{m}(t)\right)\left(\cos 2\pi f_c t + j\sin 2\pi f_c t\right)\right]$$

$$\therefore S_u(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t\right].$$

This is time domain representation of a wave containing only an upper side band.

The time domain representation of SSB modulated wave containing only lower side band is given by

$$S_L(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t + \hat{m}(t)\sin 2\pi f_c t\right].$$

$\frac{A_c}{2}$ = scaling factor ; $m(t)$ = inphase component, $\hat{m}(t)$ = out of phase component.

Single tone modulation :=

If the modulating signal contains only one frequency, then it is single tone modulation.

Consider the sinusoidal modulating signal

$$m(t) = A_m \cos 2\pi f_m t.$$

By changing phase of input signal by 90° with out changing its amplitude we get hilbert transform $\hat{m}(t)$.

$$\hat{m}(t) = A_m \cos(2\pi f_m t - 90^\circ)$$

$$= A_m \sin 2\pi f_m t.$$

We know that

$$S_u(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t\right].$$

Substitute $m(t)$ and $\hat{m}(t)$ in $S_u(t)$

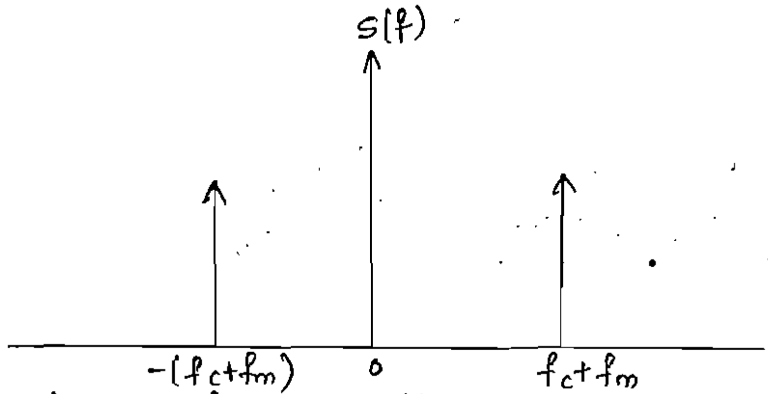
$$S_u(t) = \frac{A_c}{2}\left[A_m \cos 2\pi f_m t \cos 2\pi f_c t - A_m \sin 2\pi f_m t \sin 2\pi f_c t\right].$$

$$= \frac{A_c A_m}{2} \cos(f_c + f_m) 2\pi t$$

$$\therefore \; S_u(t) = \frac{A_c A_m}{2} \cos\left[2\pi (f_c + f_m) t\right].$$

This is SSB wave obtained by transmitting only upper side band.

Now



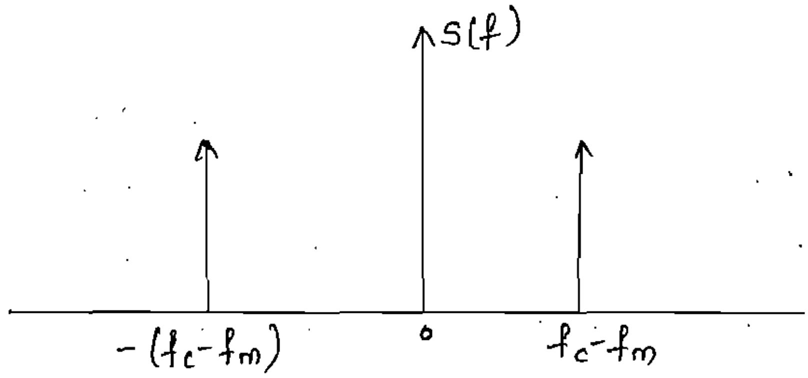
Spectrum of SSB with USB

Now for lower side band we have

$$S_L(t) = \frac{A_c}{2}\left[m(t) \cos 2\pi f_c t + \hat{m}(t) \sin 2\pi f_c t\right]$$

$$= \frac{A_c}{2}\left[A_m \cos 2\pi f_m t \cos 2\pi f_c t + \cancel{m} A_m \sin 2\pi f_m t \sin 2\pi f_c t\right].$$

$$= \frac{A_c A_m}{2}\left[\cos 2\pi (f_c - f_m) t\right].$$



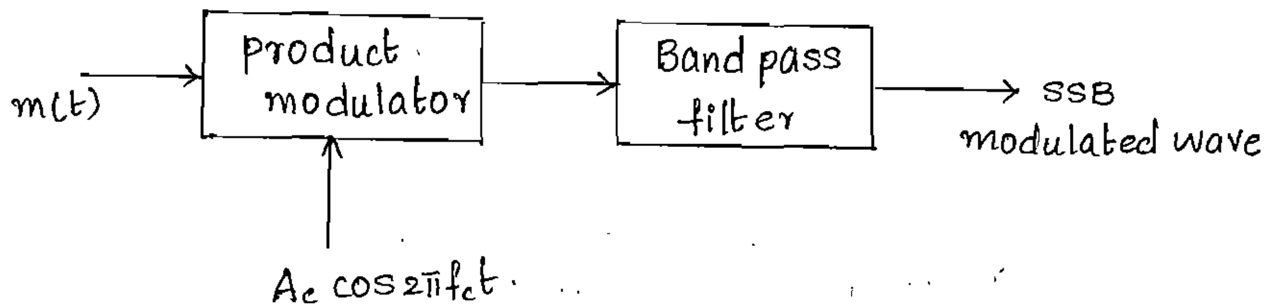
Spectrum of SSB with LSB

Generation of SSB signals :=

There are 3 different methods for generation of SSB signal.

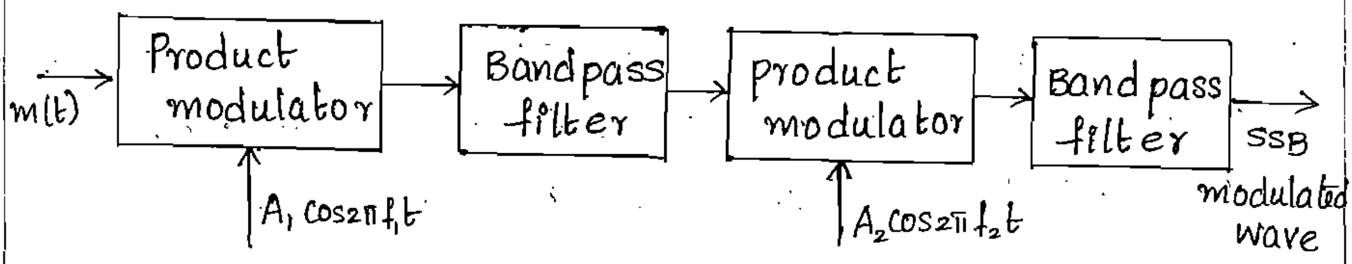
* Filter or frequency descrimination method.
* Phase shift method.
* Weavers or third method.

Filter or frequency descrimination method :=

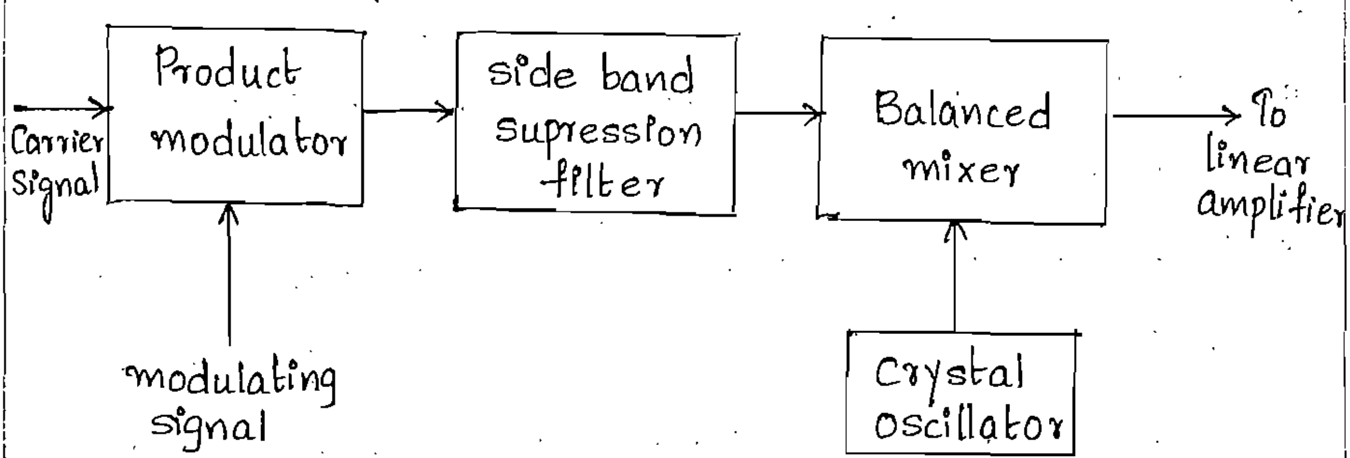


The filter is used to remove unwanted side band. The filter must have flat pass band and extremely high attenuation out side the pass band. In order to have above response the quality factor (Q) of the tuned circuits must be very high. Q-factor increases as the difference between modulating frequency f_m and carrier frequency f_c increases. Carrier frequency is usually same as transmitter frequency.

To generate an SSB modulated wave occupying a frequency that is higher than message signal, it is very difficult to design a filter, that pass desired side band and reject the other, using single stage modulation. In double stage modulation the SSB wave at the first filter is used as modulating wave for second product modulator. It produce a DSBSC wave with a spectrum that is symmetrically spaced about second carrier frequency f_2. The frequency seperation between side bands of DSBSC modulated wave is effectively twice the first carrier frequency f_1. ∴ It is easy to remove unwanted side band.



Block diagram of two stage SSB modulator.



Filter method of side band supression

For transmitting higher frequency, the Q value should be very high which can not be achieved practically. So initial modulation is carried out. The balanced modulator supress the carrier and filter supress the side band. The frequency of SSB is very low when compared to transmitter frequency This frequency is boosted up to transmitter frequency by balanced mixer and crystal oscillator. This process is called "up conversion". The side band signal is then amplified by linear amplifier.

Advantages:=

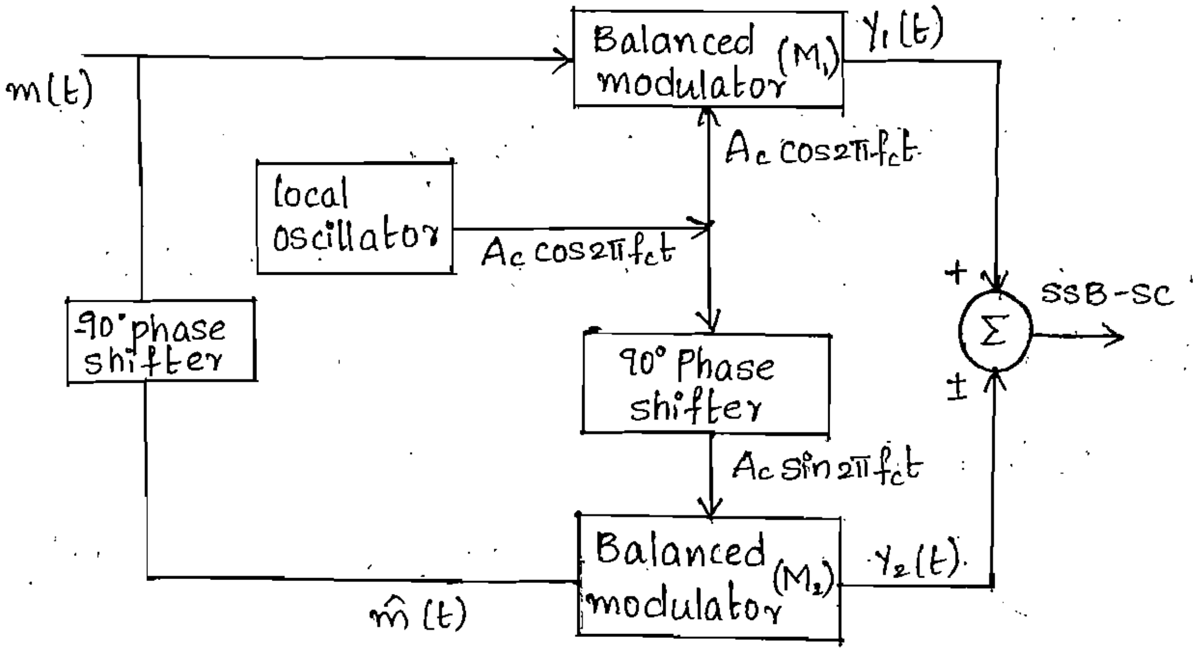
* The filter method gives side band supression up to 50 dB which is quite adequate.
* The side band filters also helps to attenuate carrier if present in output of balanced modulator.
* Band width is sufficiently flat and wide.

Dis advantages:=

* They are bulky.
* At lower audio frequencies expensive filters are required.
* As modulation takes place at lower carrier frequency repeated mixing is required in conjunction with extremely stable oscillators to generate SSB at high radio frequency.

Phase descrimination method :-

In this method we use two balanced modulators and two phase networks to obtain an SSB wave



The modulating signal m(t) and carrier signal are applied to balanced modulator (M_1) produce DSBSC wave that translates the spectrum of m(t) symmetrically spaced about the carrier frequency f_c

The modulating signal m(t) and carrier signal are phase shifted by 90° is applied to (M_2) which produce DSBSC wave with side bands of same amplitude but different phase spectra. Then we get $\hat{m}(t)$ which is hilbert transform of m(t).

The outputs of balanced modulator are identical in amplitude spectra, but phase spectra may be vector addition or subtraction.

Consider $c(t) = A_c \cos 2\pi f_c t$

$m(t) = A_m \cos 2\pi f_m t$

The output of balanced modulator M_1 is $y_1(t)$

$$y_1(t) = A_c A_m \cos 2\pi f_c t \cdot \cos 2\pi f_m t$$

$$= \frac{A_c A_m}{2}\left(\cos 2\pi(f_c - f_m)t + \cos 2\pi(f_c + f_m)t\right).$$

The input of balanced modulator M_2 is $\hat{m}(t)$.

The output is $y_2(t)$.

$$y_2(t) = A_c A_m \sin 2\pi f_c t \cdot \sin 2\pi f_m t$$

$$= \frac{A_c A_m}{2}\left(\cos 2\pi(f_c - f_m)t - \cos 2\pi(f_c + f_m)t\right).$$

Expression for lower side band is

$$y_1(t) + y_2(t) = A_c A_m \cos(2\pi(f_c - f_m)t).$$

Expression for upper side band is

$$y_1(t) - y_2(t) = A_c A_m \cos 2\pi(f_c + f_m)t.$$

Advantages:=

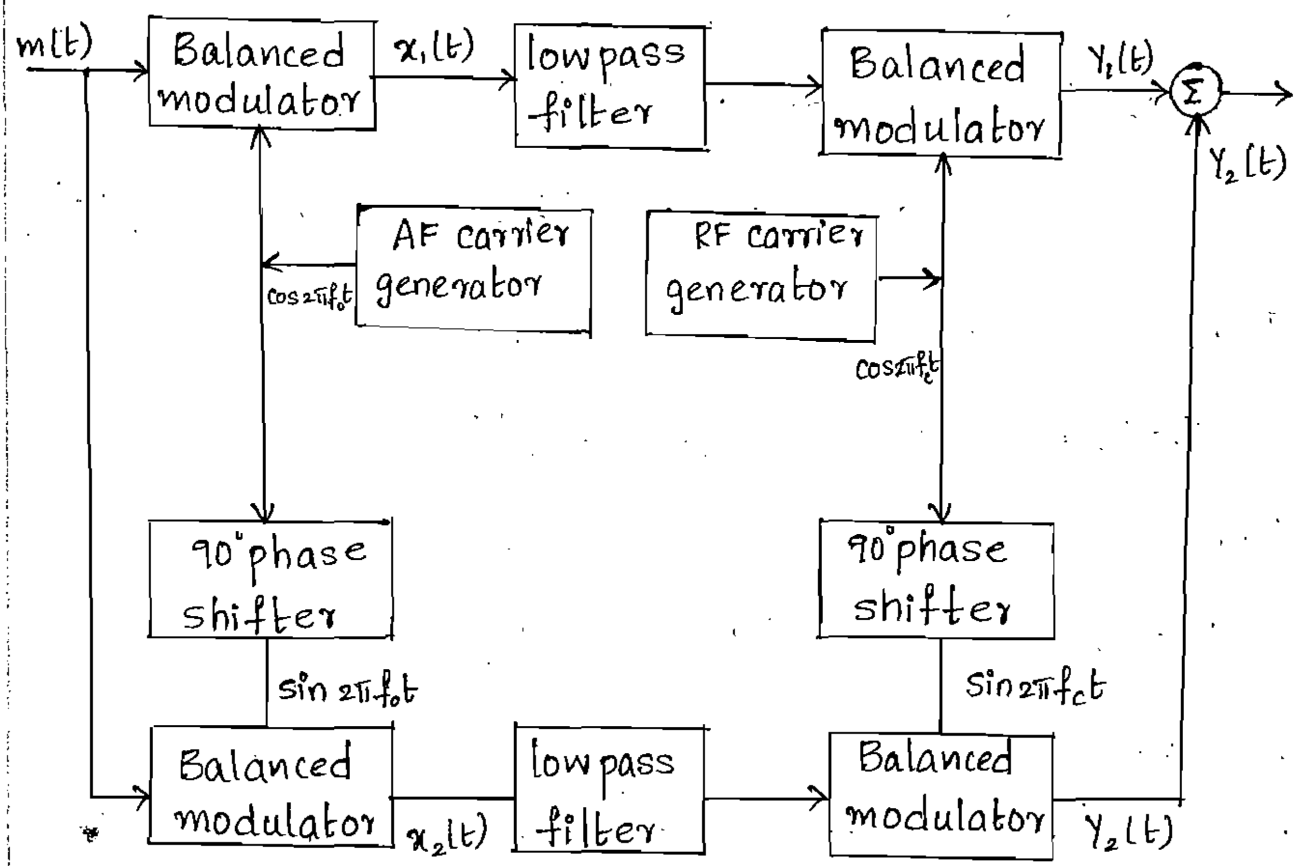
* One side band along with carrier are supressed.
* There is no need of very sharp cut off filter.
* Choice of addition or subtraction of modulator out put gives upper or lower side band.

Disadvantages:=

* The circuit is complex and expensive.
* The phase shifter should produce exact phase shift of 90°. In practise it is very difficult to design.

Weavers or third method :=

Third method of generating SSB-SC wave was invented by D.K. Weaver in 1950. This method eliminates the limitations of using sharp band pass filter. This method consists of four balanced modulators and two lowpass filters.



The input of low pass filter -1 is

$$x_1(t) = \cos(2\pi f_m t) \cdot \cos(2\pi f_0 t).$$

$$= \frac{1}{2}\left(\cos 2\pi (f_0 + f_m)t + \cos 2\pi (f_0 - f_m)t\right).$$

As it is lowpass filter it allows only low frequency i.e... $\frac{1}{2} \cos 2\pi (f_0 - f_m)t$. This is multiplied with $\cos 2\pi f_c t$ to yield $Y_1(t)$.

$$Y_1(t) = \frac{1}{2} \cos 2\pi (f_0 - f_m)t \cdot \cos 2\pi f_c t.$$

$$y_1(t) = \frac{1}{4}\left(\cos 2\pi(f_c+f_0-f_m)t + \cos 2\pi(f_c-f_0+f_m)t\right)$$

Similarly, the input of lowpass filter-2 is

$$x_2(t) = \cos 2\pi f_m t \; \sin 2\pi f_0 t$$

$$= \frac{1}{2}\left(\sin 2\pi(f_0+f_m)t + \sin 2\pi(f_0-f_m)t\right).$$

The out put is $\frac{1}{2}\sin 2\pi(f_0-f_m)t$. This is multiplied by $\sin 2\pi f_c t$ to get $y_2(t)$.

$$\therefore y_2(t) = \frac{1}{2}\sin\left(2\pi(f_0-f_m)t\right) \cdot \sin 2\pi f_c t$$

$$= \frac{1}{4}\left(\cos 2\pi(f_c-f_0+f_m)t - \cos 2\pi(f_c+f_0-f_m)t\right).$$

Expression for lower side band.

$$y_1 - y_2 = \frac{1}{2}\cos 2\pi\,(f_0+f_c-f_m)t$$

Expression for upper side band.

$$y_1 + y_2 = \frac{1}{2}\cos 2\pi(f_c-f_0+f_m)t.$$

Advantages :-

* It does not require a side band supression filter nor a wide band audio phase shift network.
* Critical parts or adjustments are not required.
* low audio frequencies can be easily transmitted.
* Side bands also be switched easily.

Disadvantages :-

* The system is more complex than other two.
* DC coupling may be needed to avoid loss of signal components close to audio carrier frequency.

Comparison between SSB generation methods.

Sl no	Parameter	Filter method	Phase shift method	Third method.
1	Method used	Filter is used to remove the unwanted side band.	Phase shifting technique is used to remove unwanted side band.	Similar to phase shift method. But carrier is phase shifted by 90°
2	90° phase shift	not required	Requires complex phase shift network.	Phase shift network is simple RC ckt
3	Possible frequency range of SSB	not possible to generate SSB at any frequency	possible to generate SSB at any frequency	possible to generate at any frequency
4	Need for up-conversion	required	not required	not required
5	Complexity	less	medium	high.
6	Design aspects	Q of tuned ckt, filter type, it size, weight and upper frequency limit	Design of 90° phase shifter for entire modulating frequency range	symmetry of balanced modulator.
7	Bulkiness	Yes	No	No
8	Switching ability	Not possible with existing circuit Extra filter and switching network is necessary	Easily possible	Easily possible. But extra crystal is required.

Demodulation of SSB waves :=

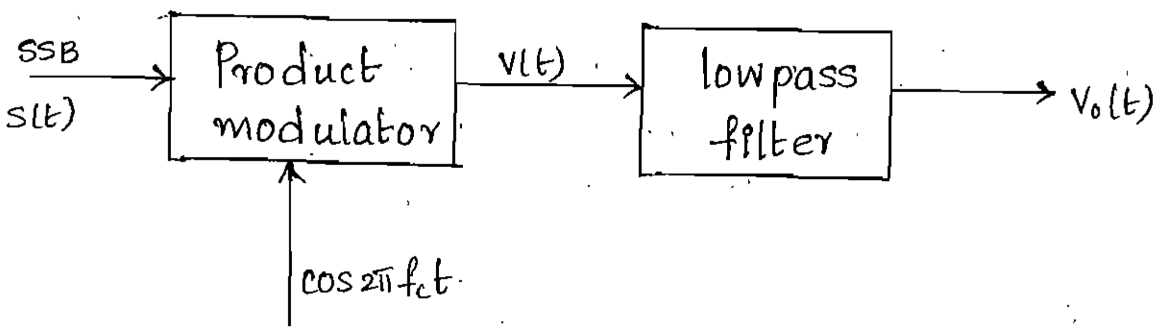
There are two types of demodulation techniques

* Coherent detection method
* Envelope detector method.

Coherent detection method:=

We can recover the message signal $m(t)$ by coherent detection method by following procedure

* Consider SSB wave either $s_u(t)$ or $s_l(t)$.
* Apply it to product modulator.
* Apply locally generated carrier $\cos 2\pi f_c t$ as second input to product modulator
* Apply output to lowpass filter. The output of low pass filter is recovered signal $m(t)$.



The output of product modulator is $v(t)$ which is product of SSB and carrier wave.

$$s(t) = \frac{A_c}{2}\left(m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t\right).$$

$\therefore V(t) = s(t) \cos 2\pi f_c t$

$$V(t) = \frac{A_c}{2} \cos 2\pi f_c t \left(m(t) \cos 2\pi f_c t \pm \hat{m}(t) \sin 2\pi f_c t \right).$$

$$= \frac{A_c}{2} m(t) \cos^2 2\pi f_c t \pm \frac{A_c}{2} \hat{m}(t) \cos 2\pi f_c t \sin 2\pi f_c t$$

$$= \frac{A_c}{2} m(t) \left(\frac{1+\cos 4\pi f_c t}{2} \right) \pm \frac{A_c}{2} \hat{m}(t) \left(\frac{\sin 4\pi f_c t}{2} \right).$$

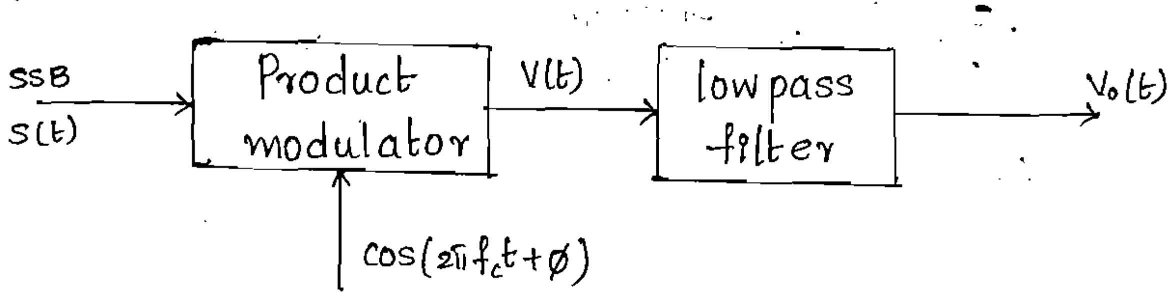
$$= \underbrace{\frac{A_c m(t)}{4}}_{\text{wanted signal}} + \underbrace{\frac{A_c}{4} \left(m(t) \cos 4\pi f_c t \pm \hat{m}(t) \sin 4\pi f_c t \right)}_{\text{unwanted signal}}.$$

If $V(t)$ is passed through low pass filter then we get

$$V_0(t) = \frac{A_c m(t)}{4}.$$

This is message signal with scaling factor $A_c/4$.

But in practise there in phase error ϕ in locally generated carrier wave.



Now $s(t) = \frac{A_c}{2} \left(m(t) \cos 2\pi f_c t \pm \hat{m}(t) \sin 2\pi f_c t \right)$

$$V(t) = s(t) \cos\left(2\pi f_c t + \phi\right)$$

$$= \frac{A_c}{2} \left(m(t) \cos 2\pi f_c t \cos(2\pi f_c t + \phi) \pm \hat{m}(t) \sin 2\pi f_c t \cdot \cos(2\pi f_c t + \phi) \right)$$

$$= \frac{A_c}{4}\left[m(t)\left(\cos(4\pi f_c t+\phi) + \cos\phi\right) \pm \hat{m}(t)\left(\sin(4\pi f_c t+\phi)+ \sin\phi\right)\right].$$

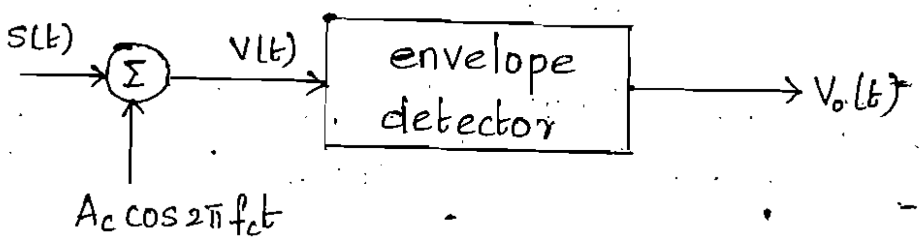
$$= \frac{A_c}{4}\left[m(t)\cos\phi \pm \hat{m}(t)\sin\phi\right] + \frac{A_c}{4}\left(m(t)\cos(4\pi f_c t+\phi) \pm \hat{m}(t)\sin(4\pi f_c t+\phi)\right]$$

$\therefore$ the desired signal

$$V_0(t) = \frac{A_c}{4}\left[m(t)\cos\phi \pm \hat{m}(t)\sin\phi\right].$$

Envelope detector method :=

In this method the SSB signal is demodulated by adding local carrier to it at the reciever and then passing the resultant wave through an envelope detector.



Consider SSB-SC with upper side band as

$$s(t) = \left(m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t\right)$$

The local carrier is given by

$$c(t) = A_c \cos 2\pi f_c t$$

The resulting signal is given by

$$V(t) = S(t) + A_c \cos 2\pi f_c t$$

$$= \left(m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t\right) + A_c \cos 2\pi f_c t$$

$$= \cos 2\pi f_c t\left(m(t) + A_c\right) - \hat{m}(t)\sin 2\pi f_c t.$$

The output of envelope detector is given by

$$V_o(t) = \sqrt{(\text{inphase component})^2 + (\text{Out of phase component})^2}$$

$$= \sqrt{(A_c + m(t))^2 + (\hat{m}(t))^2}$$

$$= \sqrt{A_c^2 + m^2(t) + 2A_c m(t) + \hat{m}^2(t)}$$

Assuming that $m^2(t) << A_c \Rightarrow \hat{m}^2(t) << A_c$

$$V_o(t) = \sqrt{A_c^2 + 2A_c m(t)}.$$

$$= \sqrt{A_c^2\left(1 + \frac{2m(t)}{A_c}\right)}$$

$$= A_c\sqrt{1 + \frac{2m(t)}{A_c}}$$

Since $|m(t)| << A_c$

$$V_o(t) = A_c\left(1 + \frac{2m(t)}{2A_c}\right) \qquad \left[\because (1+x)^{1/2} = 1 + \tfrac{1}{2}x\right]$$

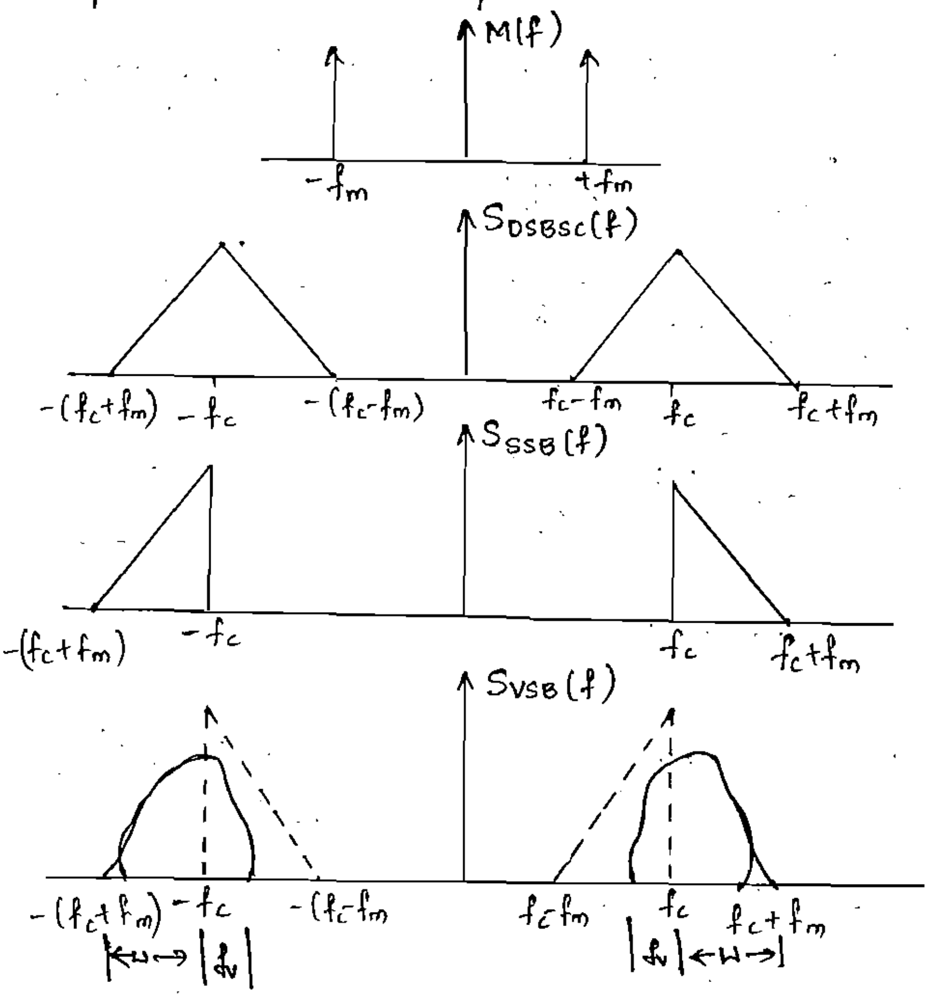
$$V_o(t) = A_c + m(t).$$

In this method local carrier frequency should match with original carrier. If not distortion occurs.

Vestigial side band modulation:=

When the message signal contains extremely low frequencies SSB modulation is not appropriate. Because the upper and lower side bands meet at carrier frequency and it becomes difficult to isolate one ~~the~~ side band. To over come this (VSB) Vestigidal side band modulation is used. In this modulation one side band is passed completely along with a trace or vestige of other side band. This technique is mostly used in television transmission.

Frequency domain description:=

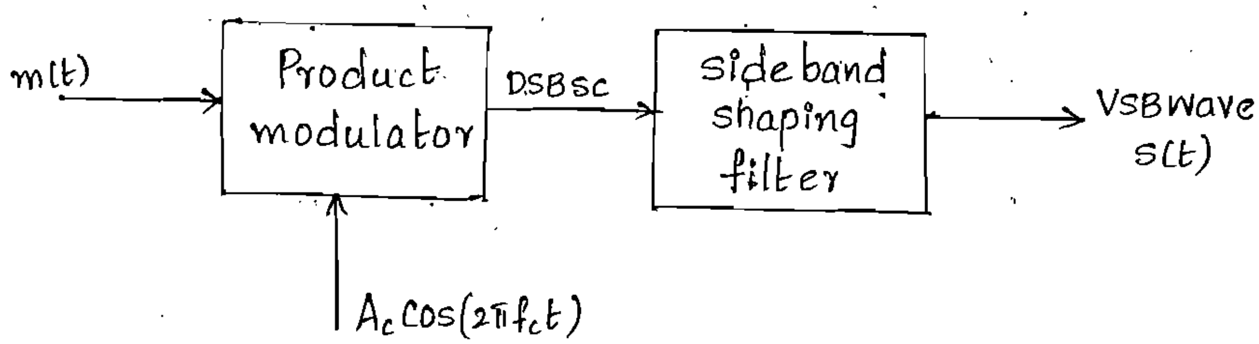


Here the lower side band is modified into vestigial side band. The transmitted vestige of LSB compensate for the removed part of USB. The transmission band width required by VSB modulated wave is given by bandwidth = $W + f_v$.

Where W is bandwidth of message signal and f_v is band width of VSB. The VSB requires bandwidth almost equal to SSB transmission. It retains the excellent low frequency base band characteristics of double side band modulation.

Generation of VSB modulated wave:-

We can generate the VSB modulated wave by passing DSBSC modulated wave through a side band shaping filter. The filter is designed to provide desired spectrum of VSB modulated wave.



The relation between the transfer function H(f) of the filter and spectrum S(f) of VSB modulated wave s(t) defined by

$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] H(f).$$

Demodulation of VSB modulated wave:=

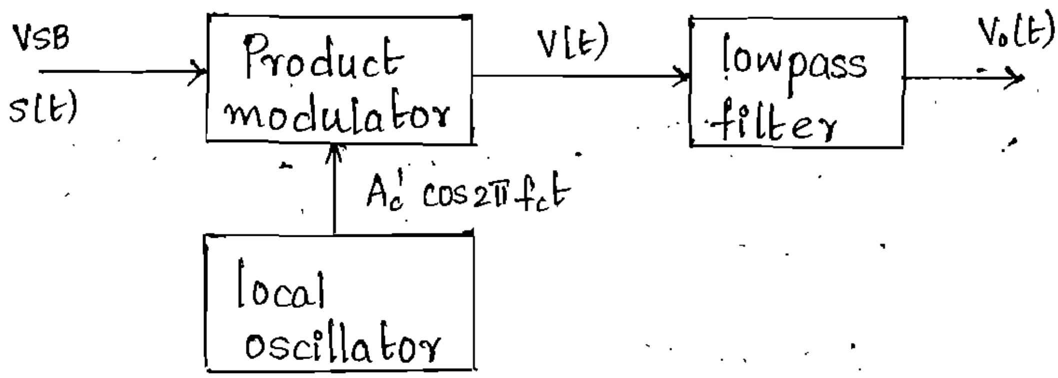
There are two methods of demodulation

* Coherent detection
* Envelope detection.

Coherent detection:=

In this method the VSB wave s(t) is passed through a coherent detector and determining the necessary condition for detector output to provide undistorted message signal m(t). In this s(t) is multiplied with locally generated wave $A_c' \cos 2\pi f_c t$. which is synchronous with carrier wave in both frequency and phase.

$$\therefore \; V(t) = s(t) \cdot A_c' \cos 2\pi f_c t$$



$$V(t) = s(t) \, A_c' \cos 2\pi f_c t$$

converting it in to frequency domain

$$V(f) = \frac{A_c'}{2} \left(S(f-f_c) + S(f+f_c) \right)$$

We have $s(f) = \frac{A_c}{2}\left(M(f-f_c) + M(f+f_c)\right) H(f)$

Substitute $s(f)$ in $v(f)$.

$$V(f) = \frac{A_c A_c'}{4}\Big[M(f-f_c-f_c) + M(f-f_c+f_c) + M(f+f_c\mp f_c) + M(f+f_c+f_c)\Big]\left(H(f-f_c) + H(f+f_c)\right).$$

$$= \frac{A_c A_c'}{4}\Big[M(f-2f_c) + 2M(f) + M(f+2f_c)\Big]\left(H(f-f_c) + H(f+f_c)\right).$$

By passing it through a low pass filter

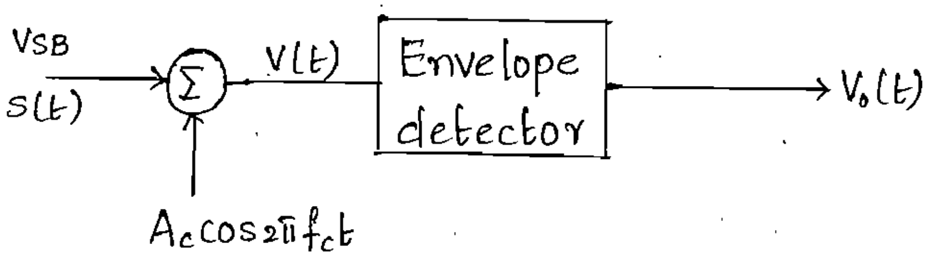
$$V_0(f) = \frac{A_c A_c'}{2} M(f)\left[H(f-f_c) + H(f+f_c)\right]$$

Condition for getting original message signal with a scaling factor and with out any distortion is given as $H(f-f_c) + H(f+f_c) = 2H(f)$.

So $V_0(f) = \frac{A_c A_c'}{2} M(f) \cdot 2H(f)$.

$V_0(f) = A_c A_c' M(f) H(f)$.

Envelope detection method :=



To make demodulation of VSB wave an envelope detector is used at the receiving end and it is necessary to transmit a carrier along with the modulated wave. The scaled expression of VSB wave by factor k_a with carrier component $A_c \cos 2\pi f_c t$ can be given by

$$V(t) = s(t) + A_c \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + \frac{A_c}{2} k_a m(t) \cos 2\pi f_c t - \frac{A_c}{2} k_a m_Q(t) \sin 2\pi f_c t$$

$$= A_c \left(1 + \frac{k_a}{2} m(t)\right) \cos 2\pi f_c t - \frac{A_c}{2} k_a m_Q(t) \sin 2\pi f_c t.$$

$$V_o(t) = \sqrt{(\text{inphase component})^2 + (\text{out of phase component})^2}$$

$$= \sqrt{\left(A_c\left(1 + \frac{k_a}{2} m(t)\right)\right)^2 + \left(\frac{A_c}{2} k_a m_Q(t)\right)^2}$$

$$= \sqrt{A_c^2 \left(1 + \frac{k_a}{2} m(t)\right)^2 + A_c^2 \left(\frac{k_a m_Q(t)}{2}\right)^2}$$

$$= A_c \left(1 + \frac{k_a}{2} m(t)\right) \left[1 + \frac{\frac{k_a}{2} m_Q(t)}{1 + \frac{k_a}{2} m(t)}\right]^{1/2}$$

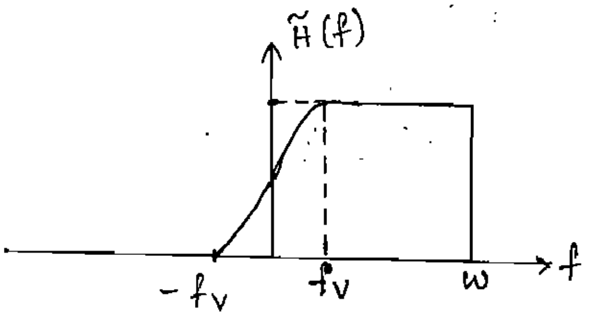
The detector output is distorted by quadrature component $m_Q(t)$. To reduce the distortion we can decrease the modulation index k_a or we can decrease the quadrature phase component $m_Q(t)$.

Time domain description :=

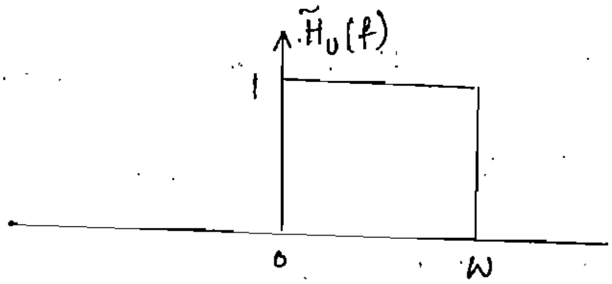
let s(t) be VSB modulated wave containing a vestige of lower side band, and it is an output of side band shaping filter in response to DSBSC modulated wave

If H(f) is transfer function of side band shaping filter then $\tilde{H}(f)$ can be given as

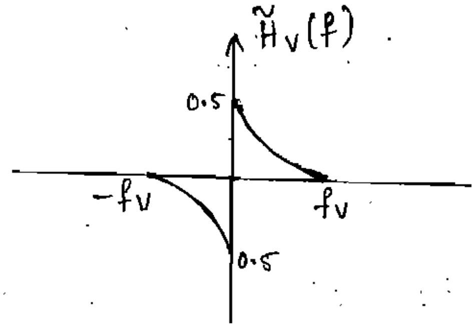
$$\tilde{H}(f) = \tilde{H}_U(f) - \tilde{H}_V(f).$$



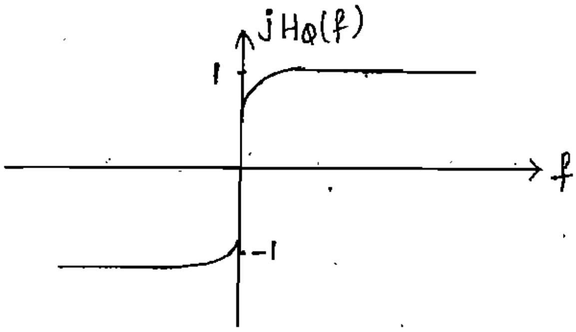
Response of side band shaping filter



first component. $\tilde{H}_U(f)$



second component $\tilde{H}_V(f)$



Response of $jH_Q(f)$

Where

(i) $\tilde{H}_v(f)$ is for complex low pass filter which is equal to the band pass filter designed to reject lower side band

(ii) $\tilde{H}_v(f)$ is for both generation of vestige of lower side band and for removal of some portion in upper side band.

We know that $\tilde{H}_v(f) = \frac{1}{2}(1 + sgn(f)) \quad 0 < f < \omega$

$= 0 \quad$ elsewhere.

$$\therefore \tilde{H}(f) = \frac{1}{2}(1 + sgn(f)) - \tilde{H}_v(f) \quad f_v < f < \omega$$

$$= 0 \quad \text{else where.}$$

As signum function and $\tilde{H}_v(f)$ are odd functions, they contain imaginary values inverse fourier transforms There exist a new expression

$$H_Q(f) = \frac{1}{j}\left(sgn(f) - 2\tilde{H}_v(f)\right).$$

$\tilde{H}(f)$ can be written as

$$\tilde{H}(f) = \frac{1}{2}(1 + jH_Q(f)) \quad f_v < f < \omega$$

$$= 0 \quad \text{else where.}$$

The VSB modulated wave $s(t)$ is given as

$$s(t) = Re\left(\tilde{s}(t)\, e^{j2\pi f_c t}\right).$$

$$\tilde{S}(f) = \tilde{H}(f)\, \tilde{S}_{DSBSC}(f)$$

$$\tilde{S}_{DSBSC}(f) = A_c M(f)$$

$$\tilde{S}(f) = \frac{1}{2} A_c \left(1 + jH_Q(f)\right) M(f).$$

Converting it in to time domain

$$\tilde{s}(t) = \frac{1}{2} A_c \left(m(t) + j m_Q(t)\right)$$

$$s(t) = \mathrm{Re}\left(\tilde{s}(t)\, e^{j2\pi f_c t}\right).$$

$$= \mathrm{Re}\left(\frac{1}{2} A_c \left(m(t) + j m_Q(t)\right)\right) \cdot \left(\cos 2\pi f_c t + j \sin 2\pi f_c t\right)$$

$$= \mathrm{Re}\Big(\frac{A_c}{2} m(t) \cos 2\pi f_c t + j \frac{A_c}{2} m(t) \sin 2\pi f_c t + j \frac{A_c}{2} m_Q(t) \cos 2\pi f_c t - \frac{A_c}{2} m_Q(t) \sin 2\pi f_c t\Big).$$

$$= \frac{A_c}{2} m(t) \cos 2\pi f_c t - \frac{A_c}{2} m_Q(t) \sin 2\pi f_c t.$$

$$\therefore\ s(t) = \frac{A_c}{2}\left(m(t) \cos 2\pi f_c t - m_Q(t) \sin 2\pi f_c t\right).$$

This is expression for USB with a vestige of LSB. Similarly the expression for LSB with a vestige of USB is given as

$$s(t) = \frac{A_c}{2}\left(m(t) \cos 2\pi f_c t + m_Q(t) \sin 2\pi f_c t\right).$$

Advantages of VSB :=

* Low frequencies, near f_c are transmitted without any attenuation.
* Band width is reduced compared to DSB.
* VSB signals are relatively easy to generate.
* VSB inherts the advantages of DSB and SSB but avoid their disadvantages at a small cost.

Comparison of amplitude modulation techniques:=

Sl no	Parameter	Standard AM	SSB	DSBSC	VSB
1	Power	High	less	medium	less than DSBSC but greater than SSB
2	Bandwidth	$2f_m$	f_m	$2f_m$	$f_m < BW < 2f_m$
3	Carrier Supression	NO	Yes	yes	NO
4	Receiver Complexity	simple	complex	complex	simple
5	Modulation type	Non linear	linear	linear	linear
6	Sideband supression	NO	One sided Completely	NO	One sideband supressed partly
7	Transmission efficiency	minimum	maximum	moderate	moderate
8	Application	Radio communication	Point to point communication Preffered for long distances	Point to point communication	Television broad casting

2- DSB & SSB Modulation :-

Introduction to DSB-SC Modulation :

The standard o/p expression for an AM wave is

$$S(t) = A\cos\omega_c t + x(t)\cos\omega_c t$$

(or)

$$S(t) = \underbrace{A\cos\omega_c t}_{\text{Carrier freq}} + \frac{Am_a}{2}\cos\underbrace{(\omega_c+\omega_m)t}_{\text{USB freq}} + \frac{Am_a}{2}\cos\underbrace{(\omega_c-\omega_m)t}_{\text{LSB frequency.}}$$

} DSB-FC.

From the above equation, we can observe that the carrier frequency component in AM wave doesnot carry any information. From the power calculations of AM wave, it has been observed that 67% of total power is required for transmitting the carrier signal which doesnot contain any information. Hence, if this carrier is supressed, only the side bands remains and in this way a saving of $\frac{2}{3}$rd power may be achieved at 100% modulation. This type of carrier supression doesnot affect the modulating signal in any way.

The resultant signal obtained by supressing the carrier from the modulated wave is called DSB-SC system (or) DSB-SC wave.

DSB-SC :-

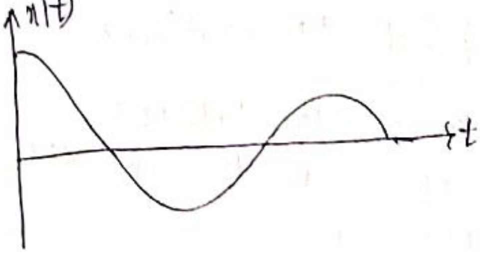
DSB-SC is a method of transmission where only the two sidebands are transmitted by supressing the carrier signal. (or) in which the carrier is The conventional AM wave suppressed is called as DSB-SC modulation.

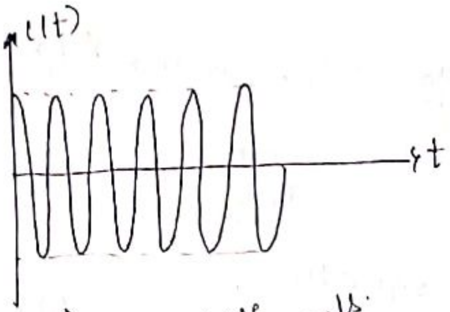
Time domain representation of Multitone DSB-SC wave :-

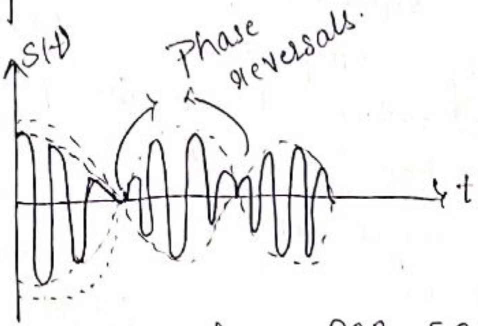
Let us consider a modulating signal $x(t)$ modulates a high frequency carrier signal. $c(t) = A\cos\omega_c t$

Let $x(t)$ be band limited to the interval $(-\omega_m, \omega_m)$

This means it doesnot have any frequency component outside the range $(-\omega_m, \omega_m)$







The standard equation for DSB-SC modulated wave is $s(t) = x(t) \cdot c(t) = x(t) A\cos\omega_c t$

Frequency domain description of DSB-SC wave :-

From the frequency shifting theorem of Fourier transform

wKT $\quad x(t) \longleftrightarrow X(\omega)$

$$x(t)\, e^{j\omega_c t} \longleftrightarrow X(\omega-\omega_c)$$

$$x(t)\, e^{-j\omega_c t} \longleftrightarrow X(\omega+\omega_c)$$

Now, $s(t) = x(t) A\cos\omega_c t$

Apply FT on above eq.

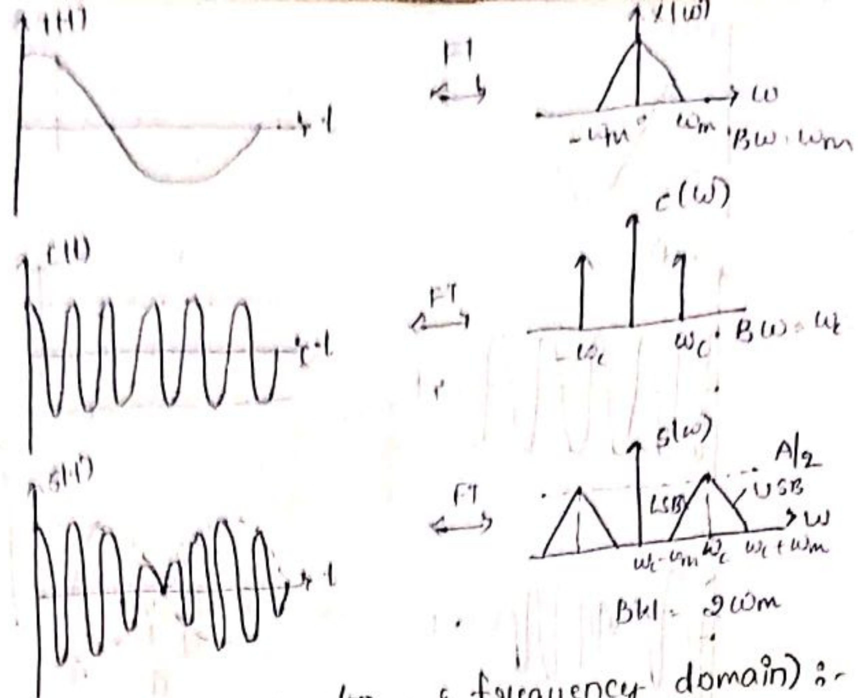
$$S(\omega) = FT\left[x(t) A\cos\omega_c t\right]$$

$$= A\, FT\left[x(t)\left(\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}\right)\right]$$

$$= \frac{A}{2} FT\left[x(t)\, e^{j\omega_c t}\right] + FT\left[x(t)\, e^{-j\omega_c t}\right]$$

$$S(\omega) = \frac{A}{2}\left[X(\omega-\omega_c) + X(\omega+\omega_c)\right]$$

→ Band width of modulating signal is ω_m



Single tone DSB-SC Modulation (Time & frequency domain):-

If the modulating signal consists of fixed frequency/single, then it is called as single tone modulation.

Let us consider a modulating signal $x(t) = A_m \cos \omega_m t$ modulates a high frequency carrier signal $c(t) = A_c \cos \omega_c t$. WKT According to the standard eq'n of DSB-SC

$s(t) = x(t) \cdot c(t)$

$= A_m \cos \omega_m t \cdot A_c \cos \omega_c t$

$= A_m A_c \cos \omega_m t \cos \omega_c t$

$= A_m A_c \frac{1}{2}\left[\cos(\omega_m + \omega_c)t + \cos(\omega_m - \omega_c)t\right]$

$$s(t) = \frac{A_m A_c}{2}\left[\cos(f_m + f_c)2\pi t + \cos 2\pi (f_m - f_c)t\right] \quad (1)$$

WKT

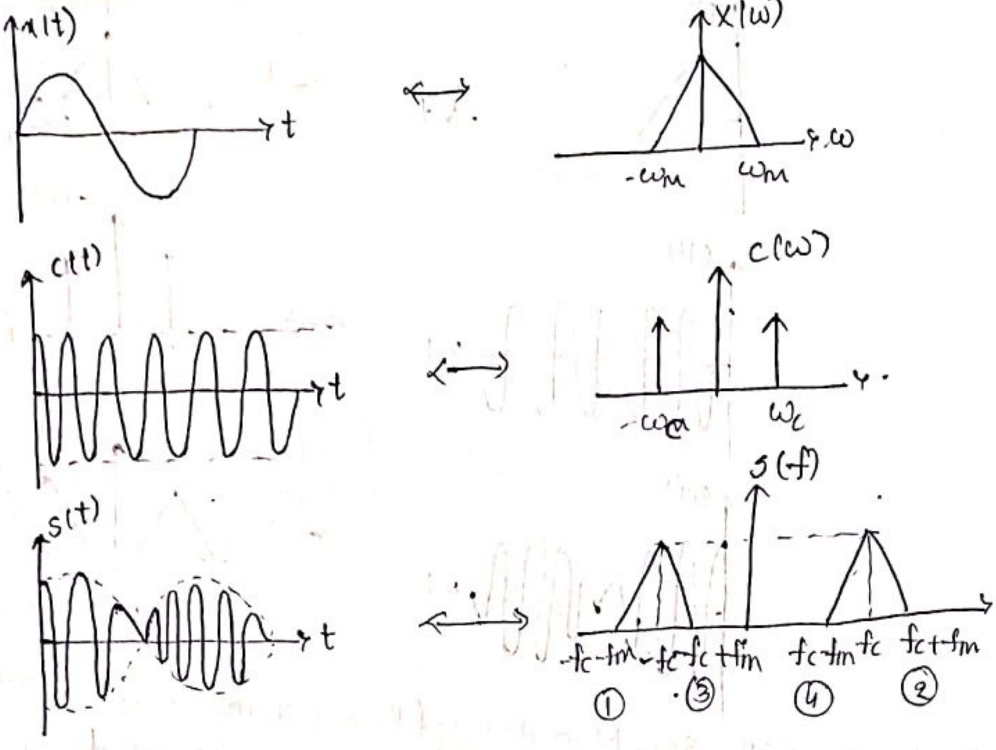
$$FT[\cos 2\pi f_0 t] = \frac{1}{2}\left[\delta(f - f_0) + \delta(f + f_0)\right]$$

Now Apply the FT on eq (1) in order to get single tone DSB-SC frequency description of

$$S(f) = \frac{A_m A_c}{2 \cdot 2}\left[\delta[f - (f_m + f_c)] + \delta(f + f_m + f_c) + \delta(f - (f_c - f_m)) + \delta(f + f_c - f_m)\right]$$

$$= \frac{A_m A_c}{4}\left[\delta(f - f_c - f_m) + \delta(f + f_m + f_c) + \delta(f - f_c + f_m) + \delta(f + f_c - f_m)\right] \quad (2)$$

The above eq. reveals four frequency components and they are upper and lower sidebands ~~of~~ on either side of $\pm f_c$

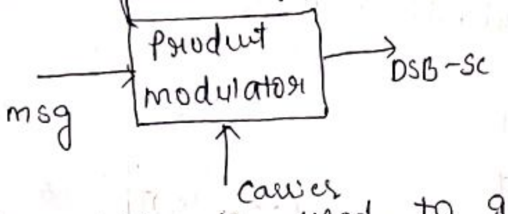


$B\omega = 2f_m$

Generation of DSB-SC :

A DSB-SC wave can be generated by simply multiplying message signal and carrier signal.

Symbolic diagram of DSB-SC generation

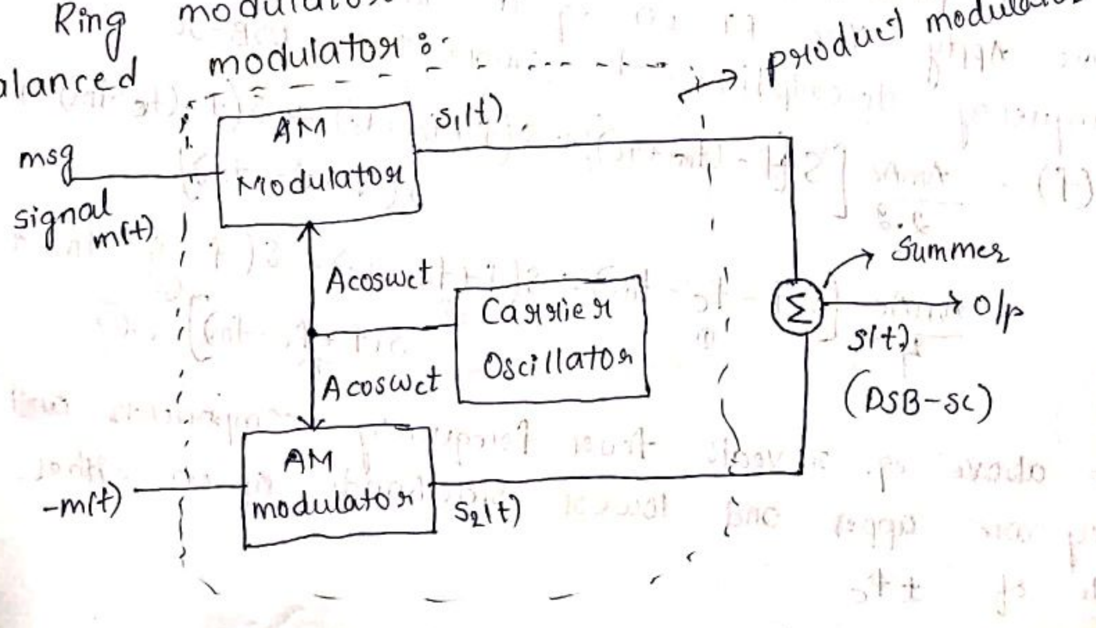


The ckt which is used to generate a DSB-SC wave by multiplying message and carrier signals is called as product modulator.

Product modulators are of 2 types. They are

1. Balanced modulator
2. Ring modulator

Balanced modulator :



$s_1(t) = A\cos\omega_c t\,[1 + m_a m(t)]$

$s_2(t) = A\cos\omega_c t\,(1 - m_a m(t))$

$s(t) = s_1(t) - s_2(t)$

$= \cancel{A\cos\omega_c t} + A m_a m(t)\cos\omega_c t - \cancel{A\cos\omega_c t} + A m_a m(t)\cos\omega_c t$

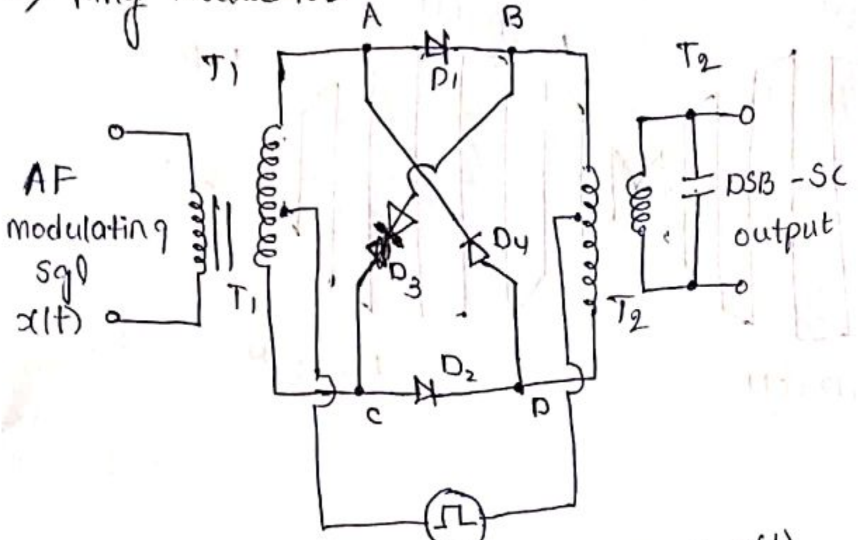
$s(t) = 2A m_a m(t)\cos\omega_c t$

(or)

$s(t) = 2m_a \underbrace{c(t)\, m(t)}$

DSB-SC wave

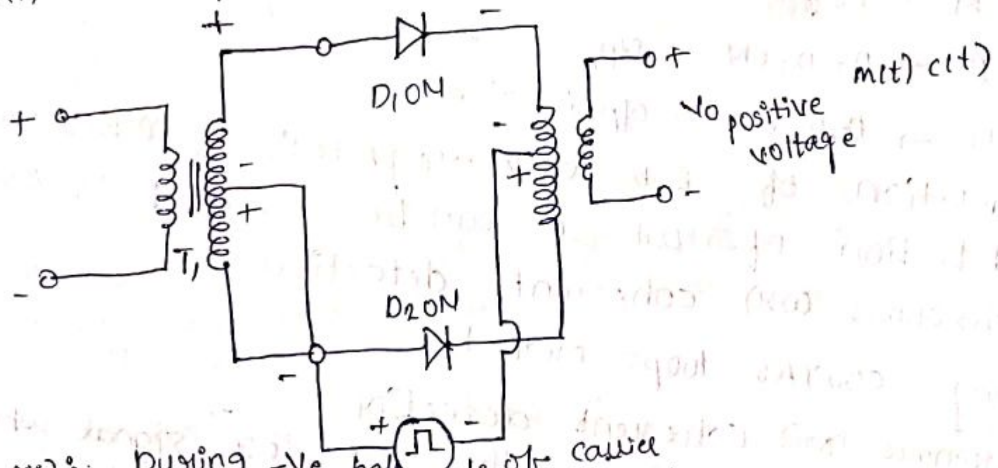
Mid. → Ring Modulator



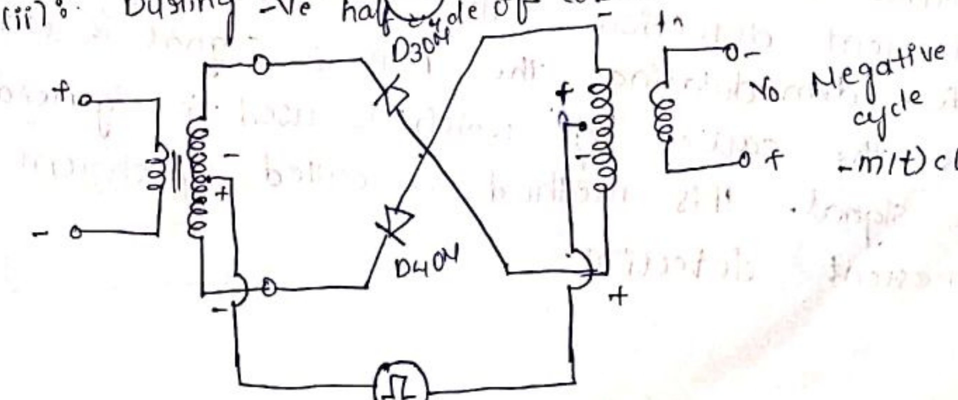
RF Carrier wave c(t)

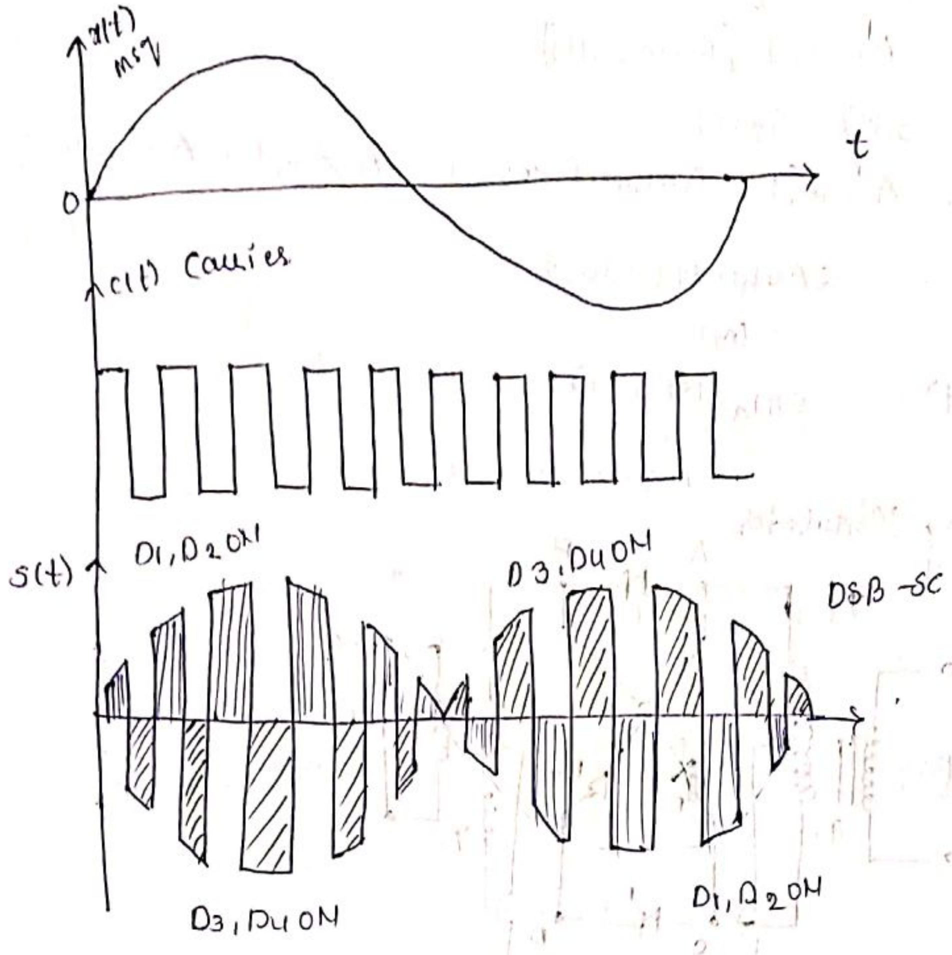
A diode ring modulator

Case (i): During +ve half cycle of carrier



Case (ii): During -ve half cycle of carrier





* *

M	C		
P	P	→ D_1, D_2 ON	- o/p voltage is +ve → $x(t)\,c(t)$
P	N	→ D_3, D_4 ON	- o/p voltage is -ve → $-x(t)\,c(t)$
N	P	→ D_3, D_4 ON	- o/p is +ve
N	N	→ D_1, D_2 ON	- o/p is -ve

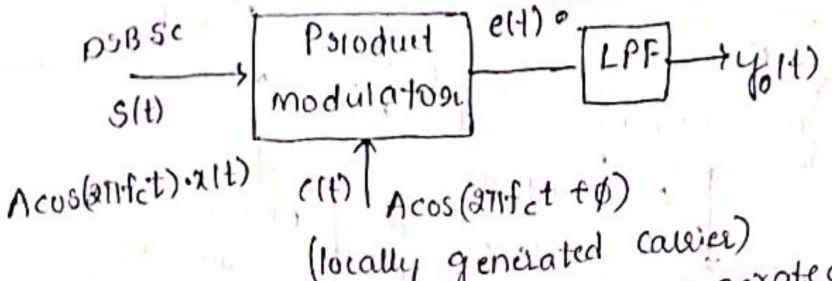
Demodulation of DSB-SC wave / Detection of DSB-SC wave:-

The detection of DSB-SC can be achieved in 2 ways.

1. synchronous (or) coherent detection
2. using COSTAS loop method.

Synchronous (or) coherent detection:-

In coherent detection, the carrier signal which is used for demodulating the DSB-SC signal is exactly same as the carrier sgl which is used for generating DSB-SC signal. This method is called synchronous (or) coherent detection.



The DSB-SC signal s(t) and a locally generated carrier sgl are applied as an inputs to a product modulator. The o/p of product modulator is $e(t) = s(t)\cdot c(t)$

$$e(t) = A\cos(2\pi f_c t)x(t)\cdot A\cos(2\pi f_c t+\phi)$$

where ϕ is phase difference b/w the carrier used at the transmitter and the receiver.

$$e(t) = A^2 x(t)\cos(2\pi f_c t)\cos(2\pi f_c t+\phi)$$

$$= \frac{A^2}{2}x(t)\left[\cos(4\pi f_c t+\phi)+\cos\phi\right]$$

$$e(t) = \underbrace{\frac{A^2}{2}x(t)\cos\phi}_{①} + \underbrace{\frac{A^2}{2}x(t)\cos(4\pi f_c t+\phi)}_{② \times}$$

Now the signal ① is allowed to pass to a LPF so 2nd term is attenuated by LPF.

$$y_0(t) = \frac{A^2}{2}x(t)\cos\phi$$

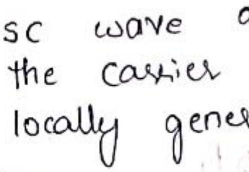
here $y_0(t)$ → demodulated signal is maximum when the phase difference $\phi = 0$

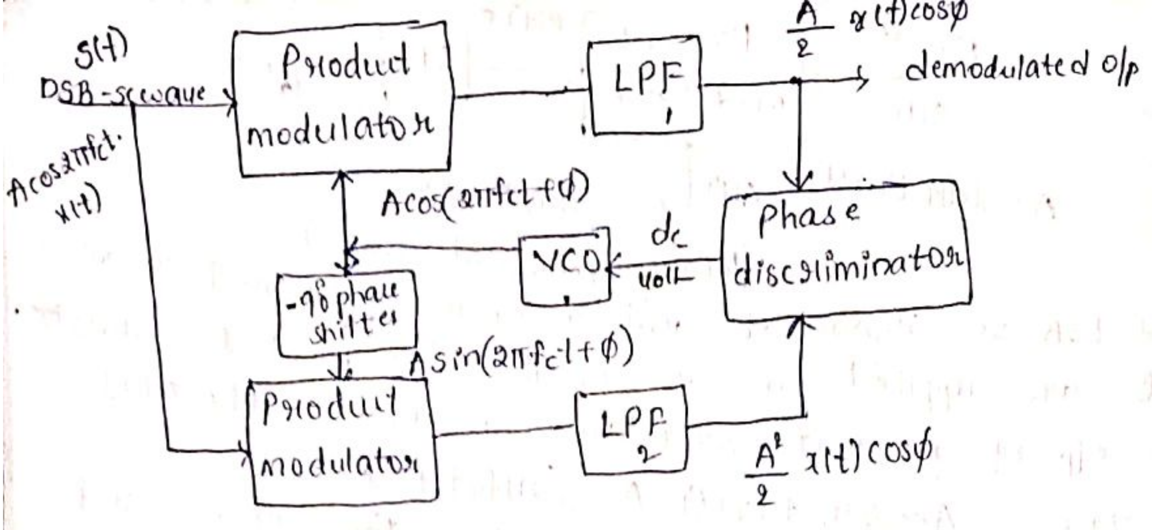
lly, when $\phi = \pm\pi/2$, the demodulated o/p is zero. This effect is called Quadrature Null effect.

COSTAS Loop Method:-

The COSTAS loop method is used for demodulation of DSB-SC wave and for the correction of face difference b/w the carrier used at transmitter section and for the locally generated signal used at the receiver section

The COSTAS Loop Method uses two product modulators with the common i/p s(t) i.e., DSB-SC wave as shown in fig. below.





VCO - Carrier generator
VCO - voltage controlled oscillator

As shown in fig, The second i/p of product modulator is taken from VCO with -90° phase shift to one of the product modulator. The o/p's of product modulators are then applied to lowpass filter ckt to allow low frequency signals. Therefore, the o/p of LPF 1 is $\frac{A^2}{2} x(t)\cos\phi$ and the o/p of LPF 2 is $\frac{A^2 x(t)}{2}\sin\phi$. Now, The o/p's of lowpass filters are applied as an inputs to phase discriminator which is used to findout the phase difference b/w these 2 signals. Depending on the phase difference b/w these 2 signals it will generate a DC voltage which is used to control the oscillations of voltage controlled oscillator by this way COSTAS loop corrects the phase difference b/w the carrier and locally generated sgl which results in error free o/p.

Power saving in DSB-SC:-

WKT, the total power transmitted by an antenna is given by $P_t = P_c\left[1+\frac{m_a^2}{2}\right]$.

For 100% modulation,

i.e., for $m_a = 1$, $P_t = P_c\left(\frac{3}{2}\right)$, $P_t = 1.5P_c$

The percentage of power saving in DSB-SC is given by

$\frac{P_c}{1.5P_c} \times 100 = 66.66\%$

i.e., 67% of power is saved for 100% modulation.

For 50% modulation, $m_a = \frac{1}{2}$.

$P_t = P_c\left[1+\frac{1}{8}\right] = 1.125P_c$

$\frac{P_c}{1.125 P_c} \times 100 = 88.88\%$

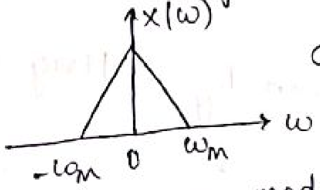
SSB-SC Modulation:

If we consider the fact that, in DSB-SC, the two side bands carries same information i.e., the Basic information is transmitted twice in DSB-SC. So, if any one of the two side bands is suppressed in DSB-SC, the resultant signal consists of single side band with suppressed carrier is called as SSB-SC wave and the modulation is called as SSB-SC modulation.

Frequency (or) spectrum representation of SSB-SC wave

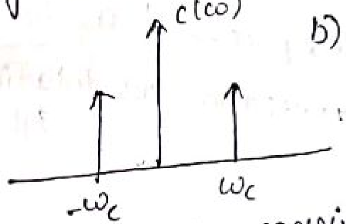
Let us consider a modulating signal $x(t)$ having the frequency range $-\omega_m$ to ω_m modulates a high frequency carrier signal $\cos\omega_c t$. Then the resulting spectrum of DSB-SC and SSB-SC are shown in the fig. below.

a)



spectrum of modulating signal

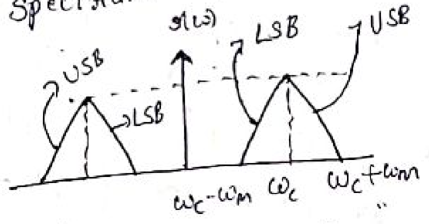
b)



Spectrum of carrier sgl

Spectrum of DSB-SC

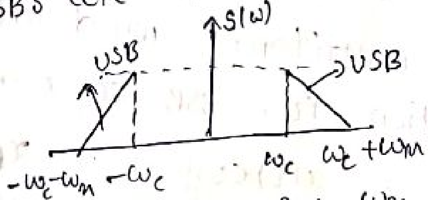
c)



$BW = 2\omega_m$

Spectrum of SSB-SC only when USB's are transmitted.

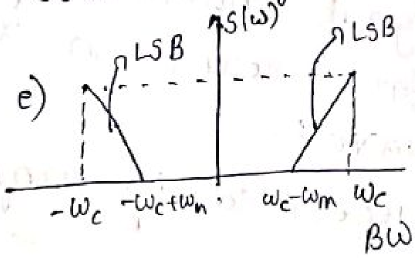
d)



$BW = \omega_m$

Spectrum of SSB-SC only when LSB's are Txed.

e)



$BW = \omega_m$

Time domain description of SSB-SC wave:

Time domain description of SSB-SC wave for single tone modulating signal

Let us consider a single tone modulating signal $x(t) = \cos\omega_m t$ modulates a high frequency carrier sgl $c(t) = \cos\omega_c t$. The time domain discription of

SSB-SC signal can be obtained with the help of fig(d), fig(e) in frequency spectrum representation.

From fig(d), it is clear that this spectrum represents the signal $\cos(\omega_c+\omega_m)t$ the time domain description of the signal

$\cos(\omega_c+\omega_m)t = \cos\omega_c t \cos\omega_m t - \sin\omega_c t \sin\omega_m t$

Similarly, from fig(e), it is clear that this spectrum of the signal $\cos(\omega_c-\omega_m)t$ corresponds to a time domain

$\cos(\omega_c-\omega_m)t = \cos\omega_c t \cos\omega_m t + \sin\omega_c t \sin\omega_m t$

From the above two expressions, the standard expression for SSB-SC wave can be expressed as

$$\boxed{s(t)_{SSB} = \cos\omega_c t \cos\omega_m t \pm \sin\omega_c t \sin\omega_m t.}$$ single tone modulating sgl

Here '+' sign represents for lower side bands and '−' sign represents for upper side bands.

Here the signal $\sin\omega_c t$ can be obtained by shifting the phase of $\cos\omega_c t$ by $-90°$ i.e.,

$\sin\omega_c t = \cos(\omega_c t - \pi/2)$.

III^{ly}, the signal $\sin\omega_m t$ can be obtained by shifting the phase of $\cos\omega_m t$ by $-90°$ i.e.,

$\sin\omega_m t = \cos(\omega_m t - \pi/2)$

For multitone modulating signal, the standard equation for SSB-SC wave can be expressed as

$$\boxed{s(t) = x(t)\cos\omega_c t + x_h(t)\sin\omega_c t}$$ Multi tone modulating sgl

$x(t) \rightarrow$ Multitone msg sgl and

$x_h(t)$ is the resultant signal obtained by shifting every frequency component present in $x(t)$.

Generation of SSB-SC wave:-

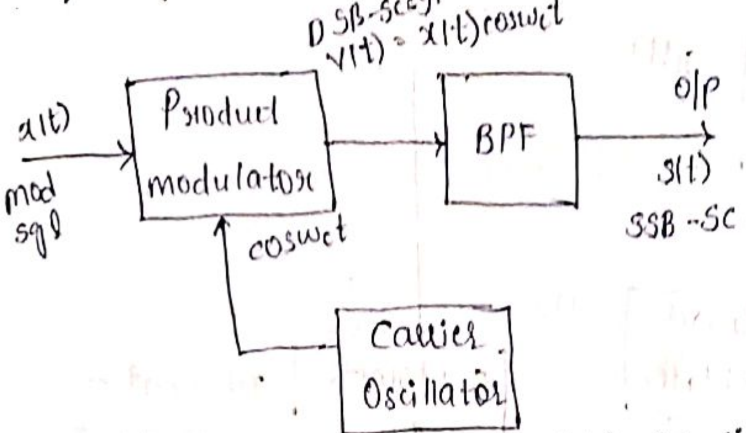
SSB-SC wave can be generated by using two methods.

They are

i) Frequency discrimination method

ii) Phase discrimination method

Frequency (or) filter discrimination method :-

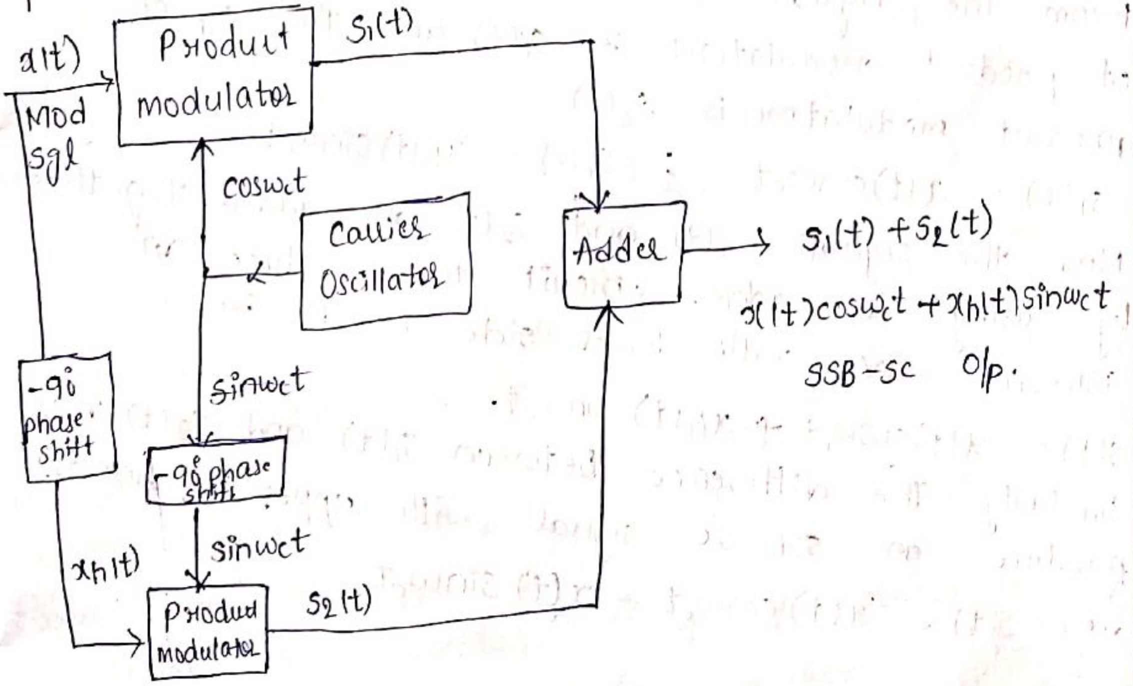


In this method, first a DSB-SC signal is generated by applying message and carrier signal as an inputs to one product modulator.

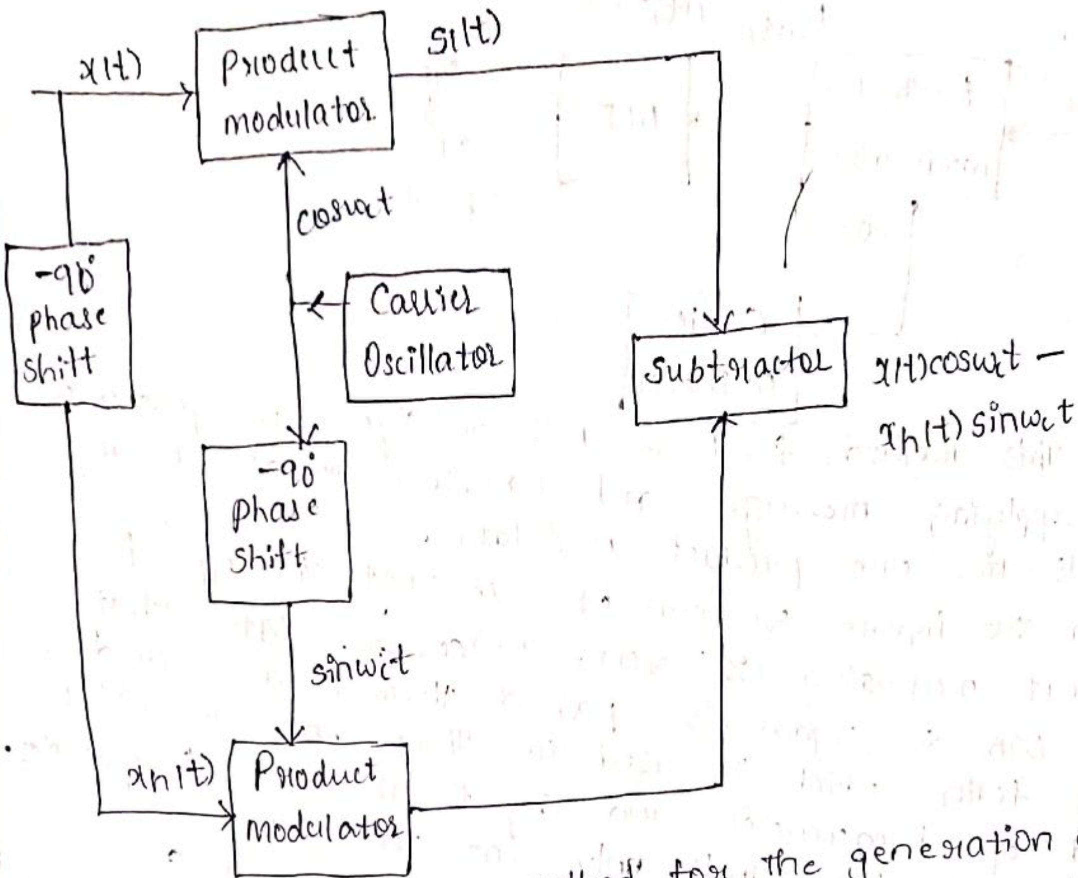
From the figure, we can observe that the o/p of product modulator is $v(t) = x(t)\cos\omega_c t$. After that the DSB-SC signal is passed through a band pass filter which is used to allow only particular band of frequencies and rejects all other frequencies. so, out of two sands only one side band is allowed by a band pass filter resulting in a SSB-SC signal generated at the o/p.

Phase discrimination Method :-

Generation of SSB-SC sgl with lower side bands :-



For Upper side bands :-

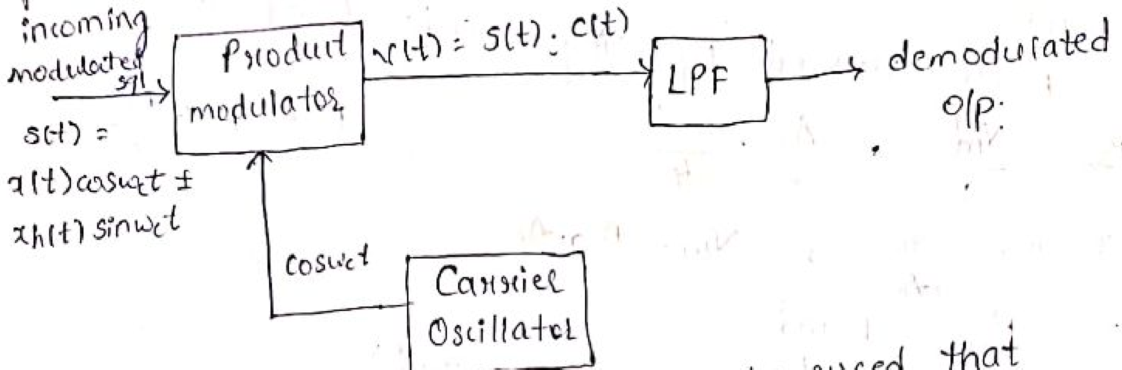


The phase discrimination method for the generation of SSB - SC uses two product modulators and two phase shifting circuits for the carrier and modulating signals respectively.
From the figure, we can observe that the output of product modulator1 is $s_1(t)$ and the o/p of product modulator is $s_2(t)$.
$s_1(t) = x(t)\cos\omega_c t$; $s_2(t) = x_h(t)\sin\omega_c t$
Now the signals $s_1(t)$ and $s_2(t)$ are added together by using an adder circuit to produce an SSB-SC wave with lower side bands. i.e.,
$s(t) = x(t)\cos\omega_c t + x_h(t)\sin\omega_c t$.
Similarly, the difference between $s_1(t)$ and $s_2(t)$ will produce an SSB-SC signal with upper side bands.
i.e., $s(t) = x(t)\cos\omega_c t - x_h(t)\sin\omega_c t$

DeModulation of SSB-SC:-

Synchronous (or) Coherent detection method:-

In synchronous detection method, the demodulated signal can be obtained by simply multiplying the incoming modulated signal s(t) with the locally generated carrier signal.



From the figure, it has been observed that s(t), the incoming modulated signal and $\cos\omega_c t$, locally generated carrier sgl are applied as an inputs to the product modulator. The o/p of product modulator is v(t). i.e., $v(t) = s(t)\cos\omega_c t$

$$v(t) = [x(t)\cos\omega_c t \pm x_h(t)\sin\omega_c t]\cos\omega_c t$$

$$v(t) = x(t)\cos^2\omega_c t \pm x_h(t)\sin\omega_c t\cos\omega_c t$$

$$v(t) = \frac{x(t)[1+\cos 2\omega_c t]}{2} \pm \frac{x_h(t)}{2}[\sin(\omega_c t+\omega_c t)+\sin(\omega_c t-\omega_c t)]$$

$$v(t) = \underbrace{\frac{x(t)}{2}}_{1} + \underbrace{\frac{x(t)\cos 2\omega_c t}{2}}_{2} \pm \underbrace{\frac{x_h(t)\sin(2\omega_c t)}{2}}_{3}$$

when this signal v(t) is passed through a low pass filter, only the ~~first term~~ first term is allowed by low pass filter and remaining terms are rejected by it. So, the o/p of LPF is nothing but the scaled version of our message sgl x(t)

Power saving in SSB-SC:-

* Calculate the percentage of power saving in SSB-SC for 100% modulation and 50% modulation.

Percentage of power saving in SSB-SC =

$$\frac{\text{carrier power} + \text{single side band power}}{\text{total power}} \quad \text{--- (1)}$$

WKT, the total power transmitted by an antenna is

$$P_t = P_c\left[1+\frac{m_a^2}{2}\right]$$

The power carried by both the side bands is

$$P_{\substack{\text{dual}\\\text{side}\\\text{bands}}} = P_S = \frac{V_m^2}{4}$$

The power carried by single side band is

$$P_{SS} = \frac{V_m^2}{8} = \frac{A_c^2}{2}\cdot\frac{m_a^2}{4}$$, here $P_c = \frac{A_c^2}{2}$

WKT $m_a = \frac{V_m}{A_c} \Rightarrow V_m = m_a \cdot A_c$

$$\therefore \; P_{SS} = \frac{P_c m_a^2}{4}$$

The % of our power $= \dfrac{P_c + P_c\left(\frac{m_a^2}{4}\right)}{P_c\left[1+\frac{m_a^2}{2}\right]} \times 100$

$$= \frac{\cancel{P_c}\left[1+\frac{m_a^2}{4}\right]}{\cancel{P_c}\left[1+\frac{m_a^2}{2}\right]} \times 100$$

For 100% modulation i.e., $m_a = 1$

$$= \frac{(1+1/4)}{(1+1/2)} \times 100$$

$$= \left[\frac{1+0.125}{1+0.5}\right] \quad \frac{\overset{0.83}{\cancel{1.125}}}{\cancel{1.5}} \times 100 = 83.3\,\%$$

For 50% modulation $\therefore m_a = 0.5$

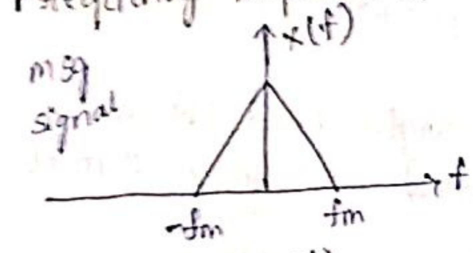
$$= \frac{\left[1+\frac{(0.5)^2}{4}\right]}{1+\frac{(0.5)^2}{2}} = 94.4\,\%$$

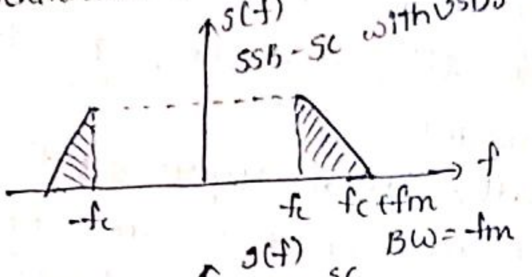
Vestigial Side band Modulation:

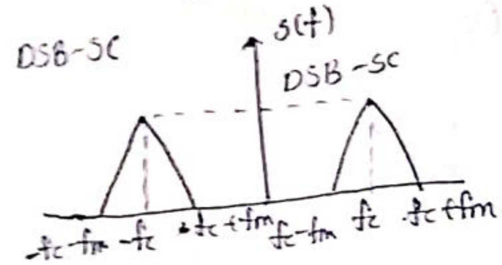
The stringent frequency response requirements on the designing of side band filter (band pass filter) in SSB-SC modulation can be relaxed by allowing some part of the other side band (vestige) to appear at the o/p of modulated ckt.

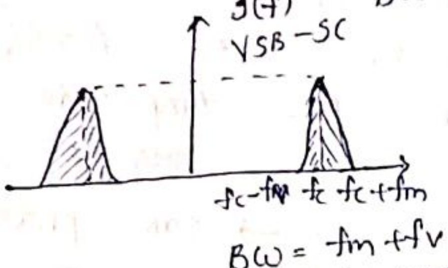
This increases the band width of the system slightly but the design of side band filter is simplified greatly. This type of modulation technique is called as VSB-SC modulation. This is the compromise b/w DSB-SC and SSB-SC modulation techniques.

Frequency Response of VSB modulation:-

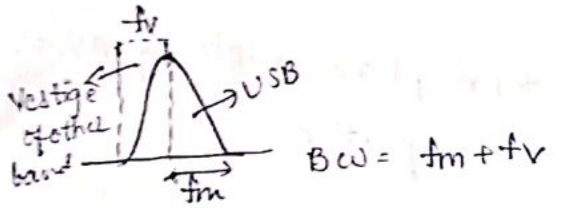








VSB modulation with upperside bands along with vestige of LSB.



Generation :-